A Decomposition Method to Optimize Concurrent Iterations Among Multiple Coupled Design Activities under Information Uncertainty

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Abstract: In this paper, an algebraic partitioning method is proposed to make a trade-off between sequential and concurrent iterations among coupled activities. First, a proposed binary variable matrix named Iteration Transition Matrix (ITM) is developed to decouple multiple interdependent activities into a number of individual pairs. The innovative aspect of the ITM variable is its application in an Integer Linear Programming (ILP) model to build the equations of constraints which represents the required iterations to accomplish coupled activities. This model contributes to estimate unknown number of sequential iterations between each pair utilizing a stationary Markov Chain (MC). These estimated numbers are assumed to be used in equation of constraints in the ILP model. Finally, after establishing the objective function, the final results of the ILP represent optimum numbers of concurrent and sequential iterations. At the end, the developed model is applied in an example of an anti-corrosion tape product development process.

Keywords: iteration transition matrix, sequential iteration, concurrent iteration, coupled activities

1 Introduction

Product development process is an iterative improvement process that will be continued until reaching the expected results (Eppinger et al., 1994). There are three types of Independent, dependent and interdependent relationships among coupled design activities. To reduce the complexity of the product development projects (Steward, 1981) developed a tool model named Design Structure Matrix (DSM). DSM is capable of illustrating interdependency among design activities.

Generally, design activities are categorized into three types of independent, dependent, and coupled activities (Eppinger and Browning, 2012). Among them, coupled activities increase the complexity (Browning and Ramasesh, 2007), (Hoedemaker et al., 1999). In addition, it is believed that coupled activities are the sources of increasing product development time by increasing the number of information exchange or iterations during accomplishing product development projects (Unger and Eppinger, 2011), (Browning, 2001), (Loch and Terwiesch,1998). Uncertainty among dependent activities is the main source to create interdependency relationships among activities (Levardy and Eppinger, 2009), (Loch et al.,2001). The information uncertainty might affect the established relationships among product development activities (Khastehdel et al., 2018), (Khastehdel and Mansour, 2012) through receiving new information from upstream to downstream activities and vice versa. Thus, a trade-off between extending development time and reducing uncertainty should be considered (Srour et al, 2013). Similarly, iterations among coupled activities are classified into two different categories including sequential and overlapping iterations (Yassine and Dan, 2003). In sequential iterations, information transfer from one activity to the other one; however, overlapping iterations might create interdependency.

(Yang et al.,2012) developed an overlapping method where there is dependency among activities. Also, (Yang et al., 2014) developed a simulation model to find the optimum degree of overlapping between sequence activities. However, these models did not consider the same issues when there is interdependency among activities. (Joglekar et al., 2001) developed a performance generation (PMG) model to optimize sequential, concurrent and overlapped strategies between two coupled design activities. However, it was not determined how the PMG works in face of multiple coupled activities. (Zhang et al., 2014) measured the degree of strength among coupled design activities in order to find the best sequence among them. Also, (Smith and Eppinger, 1997a) developed a Work Transition matrix (WTM) to measure the degree of dependency among coupled design activities to determine the convergence of iterations and optimize the sequence among coupled design activities. However, overlapping among coupled design activities was not considered. (Wang and Lin, 2008) developed a simulation-based model to find the optimum overlapping between product development activities using DSM. However, the information uncertainty parameter related to the iteration probability between coupled activities was not considered. (Yin et al., 2019) developed a model based on value analysis to optimize overlapping iterations; however, the constraints of resources and deadlines are not included. The main distinction of this research is including both sequential and concurrent iterations in an optimization process. It is assumed that measuring the number of sequential and concurrent iterations simultaneously result in determining the degree of overlapping between two coupled activities. In addition, the strength of interdependency among coupled activities is measured through estimating the length of sequential iterations among coupled activities using transition matrix. (Browning, 2015) classified a collection of research focused on decomposing coupled blocks. For example, (Martínez et al., 2011) re-arranged row and columns of coupled activities in a DSM to reduce feedback and blocks including coupled activities to improve the project performance. (Ahmadi, et al., 2001) developed an optimization model to reduce iterations using Markov chain. However, the transition between sequential and concurrent iterations is not included in the transition matrix. In this paper, the transition between sequential and concurrent iterations is taken into consideration using the ITM. (Eppinger et al., 1996) developed a model to consider parallel and sequential iterations using a probability model to improve project performance. However, the constraints of resources and overlapping iterations are not considered.

In this paper the complexity of the product development processes is reduced through a decomposition method to decouple interdependent activities. In Fig. 1 concurrent and sequential iterations between three coupled activities are illustrated through an ITM which is a DSM with binary random variable in diagonal entries and probability of iterations in off-diagonal cells. In addition, the ITM can be decomposed from higher dimension matrixes to the smaller ones. (Smith and Eppinger,1997b) developed a predictive model of sequential iteration using reward Markov Chain which is the source of initial idea to develop the ITM. However, it was assumed that only one task can be performed at a time and the total time of the coupled activities is equal to the sum of sequential iterations time. In the ITM, concurrent and sequential iterations among coupled activities is taken into consideration which is applied subsequently as the basis vectors to develop an ILP model to optimize the number of concurrent iterations. As an example, in the Figure1 three are three coupled activities decomposed to three pairs of coupled activities. The diagonal entry of (1,1) represents concurrent iteration as well as the entries of (1,0) and (0,1) illustrate sequential iterations.

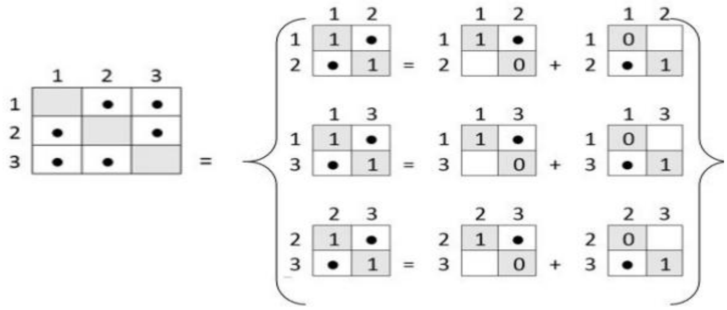


Figure 1. Decoupling of three coupled activities to concurrent and sequential iterations.

There are assumptions to use the ITM as is following:

Assumption1- If the sum of the binary values is greater than 1 it shows concurrent iterations and if the sum of them is equal 1 represents sequential iterations.

Assumption2-The ITM can be decomposed to lower dimension matrixes according to the Figure 1. This feature is important to optimize iterations through an ILP model since the ITM is utilized as the basic vectors to constitutes equations of constraints in an ILP model. This feature will be described completely in the next section.

Assumption 3- Each individual pair belongs to a specific ITM.

Assumption4-Non- diagonal entries represent the repetition probabilities between two coupled activities. The Figure 2 illustrates probability of iterations among three coupled activities with corresponding ITM's which led to three pairs of coupled activities.

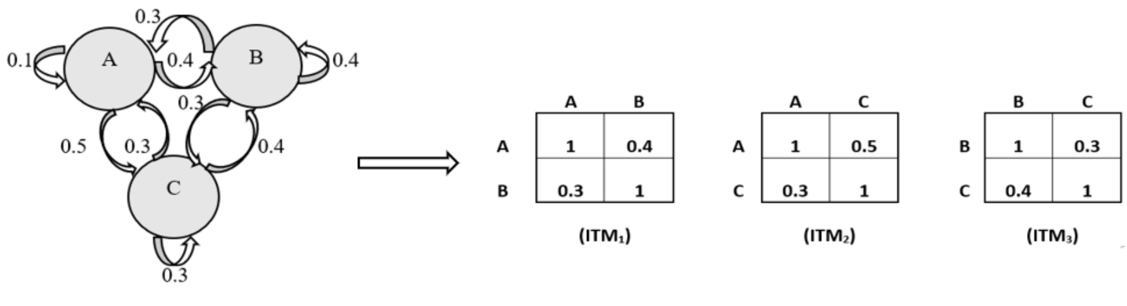


Figure 2 .Illustrating ITM's for three coupled activities with probability of iterations.

Then the unknown number of sequential iterations (n_a and n_b) between two coupled activities is estimated according to the repetition probability within a transition matrix illustrated in Figure 3.

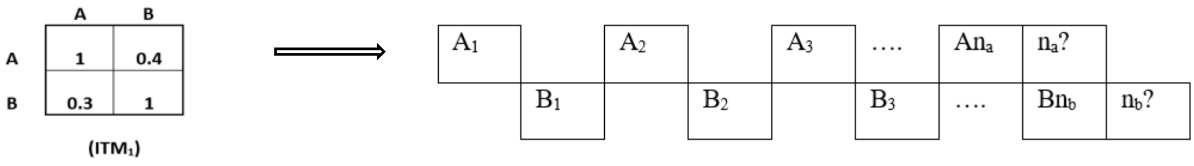


Figure 3. Illustrating the unknown numbers of sequential iterations

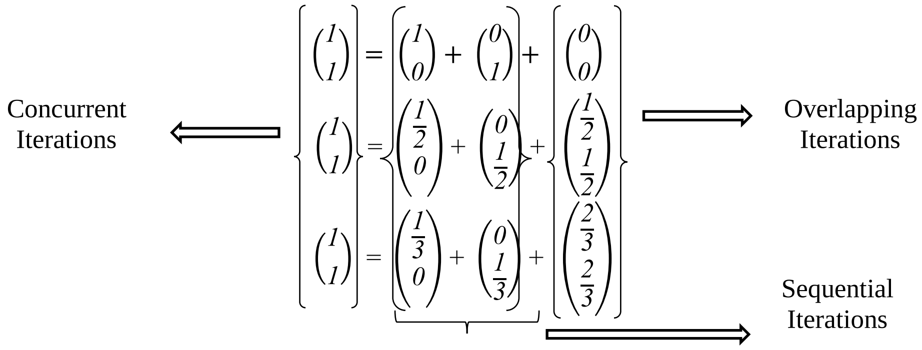
it should be noted a concurrency option including reworking is taken into consideration during transition between decomposed activities. Each ordered pair of (Ai, Bj) in the Figure 3 represents one iteration which can involve sequential and concurrent iterations for being accomplished. The Equation 1 represents a sequential iteration and the second Equation include concurrent iteration with a possibility of reworking.

$$ITM\ (Ai, Bj) = ITM\ (1,0) + ITM\ (0,1) \qquad (1)$$

$$ITM\ (Ai, Bj) = ITM\ (1,1) + Pr(reworking).\ (ITM\ (1,0) + ITM\ (0,1)) \qquad (2)$$

1-1 Defining the overlapping iterations

The focus of this research is the sequential and concurrent iterations among coupled activities. However, including overlapping iterations through the ITM is explained in this section. But, these variables excluded from the optimization model of this paper in order to keep the model with linear programming conditions. Thus, including overlapping iterations remains for future works. The Equation 3 represents the relations among sequential, concurrent and overlapping iterations for two coupled activities in the ITM.



$$\begin{pmatrix}1\\1\end{pmatrix} = k.\left(\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\right) + (1-k)\begin{pmatrix}1\\1\end{pmatrix} \qquad 0 \le k \le 1 \qquad (3)$$

Assumption5-In this paper k=1 is assumed.

2 Estimating the unknown number of sequential iterations between two coupled activities.

In this section n_a and n_b illustrated in Fig.3 are estimated using MC. Many systems have the property that the past states independent of the future states and knowing the present state is enough. This property is called the Markov property, and systems having this property are called Markov chains (Hoel and Stone,1972). The Markov property can be defined precisely by the Equation4.

$$P\ (X_{n+1}= x_{n+1} / X_0 = x_0,\ \dots,\ X_n = x_n) = P\ (X_{n+1}= x_{n+1} \mid X_n = x_n) \qquad (4)$$

The conditional probability $P\ (X_{n+1}= x_{n+1} \mid X_n = x_n)$ is called the transition probability of the chain.

Assumption6- In this study, it is assumed that the system has stationary transition probabilities that means the equation 3 is independent of n. To model the MC, each transition is representative of one iteration between design activities.

Step1- Setting the transition matrix illustrated in the Figure 4 and the initiate probability conditions are according to the Equations of 5 and 6.

$$q_1 + p_2 = 1 \qquad (5)$$

$$p_1 + q_2 = 1 \qquad (6)$$

	A	B
A	q_1	p_2
B	p_1	q_2

Figure 4. Transition probabilities between two coupled activities

$P\ (X_{A,B}) = p_2$ represents the transition probability from the activity A to the activity B.

$P\ (X_{B,A}) = p_1$ represents the transition probability from the activity B to the activity A.

$P\ (X_{A,A}) = q_1$ represents the transition probability from the activity A to B complementary (B^C).

$P\ (X_{B,B}) = q_2$ represents the transition probability from the activity B to A complementary (A^C).

Assumption7- A^C and B^C represent the probability of closing sequential iterations.

Step 2- Estimating the unknown numbers of sequential iterations between A and B.

Sequence number of Iterations	1	2	3	4	5	6	7	….	N
Iterations of activity A	A		A		A		A	…	A
Probability of Iterations of A	1	B	$P(X_{B,A,A})$	B	$P(X_{B,A,B,A,A})$	B	$P(X_{B,A,B,A,B,A,A})$	…	$P(X_{B,A,B,A,B(n-3),A(n-3),A})$
Expected Iterations of A	1	1	$3.P(X_{B,A,A})$		$5.P(X_{B,A,B,A,A})$		$7.P(X_{B,A,B,A,B,A,A})$	…	$N.P(X_{B,A,B,A,B(n-3),A(n-3),A})$

Figure 5. Illustrating the sequence number of iterations with probability of iterations for two coupled activities.

The Figure 5 represent the calculations of probability and expected iterations within each sequence number of the activity A. The expected iterations for an activity is indicated in the Equation 7.

Expected Iterations of A= (sequence number of iterations) × (probability of iterations of A) (7)

As well as, the Equations of (8,9), (10,11), (12,13) calculate the expected number of iterations within the sequence number of 3,5 and 7 respectively.

$$\begin{cases} P(X_{B,A,A}) = P(X_{B,A}).P(X_{A,A}) = p_2.q_1 & (8) \\ E(X_{B,A,A}) = 3.p_2.q_1 & (9) \end{cases}$$

$$\begin{cases} P(X_{B,A,B,A,A}) = P(X_{B,A}).P(X_{A,B}).P(X_{B,A}).P(X_{A,A}) = p_2.p_1.p_2.q_1 = p_2^2.p_1.q_1 & (10) \\ E(X_{B,A,B,A,A}) = 5.p_2^2.p_1.q_1 & (11) \end{cases}$$

$$\begin{cases} P(X_{B,A,B,A,B,A,A}) = P(X_{B,A}).P(X_{A,B}).P(X_{B,A}).P(X_{A,B}).P(X_{B,A}).P(X_{A,A}) = p_2.p_1.p_2.p_1.p_2q_1 = p_2^3.p_1^2.q_1 & (12) \\ E(X_{B,A,B,A,B,A,A}) = 7.p_2^3.p_1^2.q_1 & (13) \end{cases}$$

Then, the Equation of 14 is founded to estimate the expected sequence number of iterations as is following:

$$E(X_{B,A,\dots,Bn,An,A}) = (2n+1).p_2^n.p_1^{n-1}.q_1 \qquad (14)$$

Finally, the Equations of 15 and 16 are extracted to calculate the expected sequence number of activities of A and B respectively.

$$\begin{cases} E(x_{(A)}) = 1 + \sum_{n=1}^{n}(2n+1).p_2^n.p_1^{n-1}.q_1 & (15) \\ E(x_{(B)}) = 1 + \sum_{n=1}^{n}(2n+1).p_1^n.p_2^{n-1}.q_2 & (16) \\ 0 \le q,p \le 1 & (17) \end{cases}$$

To solve the Equations of 15 or 16, it is assumed that $p_1=p_2$.

Assumption8- $p_1=p_2$

With this limited assumption the Equation 15 is substituted by the Equation 18.

$$E(x_{(A)}) = 1 + \sum_{i=1}^{n}(2n+1).p^{2n-1}.q_1 \qquad (18)$$

The method to solve the Equation18 is represented as is following:

$$\begin{cases} E(x_{(A)}) = 1 + q_1.\sum_{n=1}^{n}(2n+1).p^{2n-1} = 2+q_1.(3p+5p^3+7p^5+9p^7+\dots = 1+\frac{q_1}{p}.(3p^2+5p^4+7p^6+9p^8+\dots) \\ = 1 + \frac{q_1}{p}.\frac{d}{d_p}(\int_0^y 3p^2 .d_p + \int_0^y 5p^4 .d_p + \int_0^y 7p^6 .d_p + \int_0^y 9p^8 .d_p + \dots + \int_0^y (2n+1).p^{2n} .d_p) \\ \text{With: } p=y \qquad (19) \\ = 1+\frac{q_1}{y}.\frac{d}{d_y}(y^3+y^5+y^7+y^9+\dots+y^{2n+1}) \\ = 1+\frac{q_1}{y}.\frac{d}{d_y}y^3.(1+y^2+y^4+y^6+\dots+y^{2n-2}) = 1+\frac{q_1}{y}.\frac{d}{d_y}y^3.\left(\frac{1}{(1-y^2)}\right) = 1+q_1.3y.\left(\frac{1}{(1-y^2)}\right)+\left(\frac{2y}{(1-y^2)^2}\right) = \text{(with p=y)} \\ = 1+q_1.\left(\left(\frac{3p}{(1-p^2)}\right)+\left(\frac{2p}{(1-p^2)^2}\right)\right) \qquad (20) \end{cases}$$

The final answer is reached according to the Equation 20.

The expected number of closing sequential iterations for the activities of A and B are according to the 21 and 22 Equations respectively. Finally, the Equation 23 determines the expected numbers of sequential iterations between two coupled activities.

$$E\left(x_{(A)}\right)=1+q_1.\left(\left(\frac{3p}{(1-p^2)}\right)+\left(\frac{2p}{(1-p^2)^2}\right)\right) \qquad (21)$$

$$E\left(x_{(B)}\right)=1+q_2.\left(\left(\frac{3p}{(1-p^2)}\right)+\left(\frac{2p}{(1-p^2)^2}\right)\right) \qquad (22)$$

Note1- It should be noted if $p_1=p_2$ then $q_1=q_2$ and the above equations is replaced by the Equation 23.

$$E\left(x_{(AB)}\right)=E\left(x_{(A)}\right)+E\left(x_{(B)}\right)=2+2q.\left(\left(\frac{3p}{(1-p^2)}\right)+\left(\frac{2p}{(1-p^2)^2}\right)\right) \qquad (23)$$

Analyzing the Equation 23 is taken into consideration through an example in the next section.

2-1 Estimating the length of sequential iterations among coupled activities through an example.

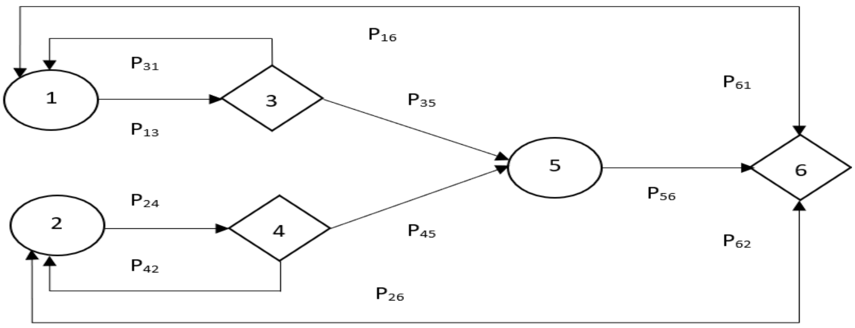


Figure 6. Development process of an anti-corrosion tape product.

The Figure 6 illustrates an example of stage-gate development process of an anti-corrosion tape product with following activities: Activity 1- Developing polyethylene tapes (the first Semi-finished product). Activity 2- Developing adhesive (the second Semi-finished product). Activity 3- Testing results of activity 1. Activity 4- Testing results of activity 2. Activity 5- Developing primer and final product. Activity 6- Testing the final product.

$P_{i,j}$- Represent repeat probability between two stages.

Step1- Illustrating the process through DSM and Determining ITM's.

		1	2	3	4	5	6
		A	B	C	D	E	F
1	A			●			●
2	B				●		●
3	C	●					
4	D		●				
5	E			●			
6	F	●	●			●	

		1	2	3	4	5	6
		A	B	C	D	E	F
1	A			ITM1			ITM3
2	B				ITM2		ITM4
3	C	ITM1					
4	D		ITM2				
5	E						
6	F	ITM3	ITM4				

Figure.7. Identifying the ITM variables in the DSM

According to the Figure 7 the expected number of sequential iterations among the five coupled activities of (A, B, C, D, F) is calculated through the Equation 24 in which four pairs of coupled activities including (A, C), (B, D), (A, F) and (B, F) are identified.

$$M=E\left(x_{(AB)}\right)+E\left(x_{(BD)}\right)+E\left(x_{(AF)}\right)+E\left(x_{(BF)}\right) \qquad (24)$$

M is the expected length of sequential iterations for the four coupled activities.

Step2- Establishing the transition matrix for the development process illustrated in Figure 8. The iteration probability values related to the (A, C) and (B, D) coupled activities extracted from the transition matrix illustrated in the Figure 9 and Figure 10.

		1	2	3	4	5	6
		A	B	C	D	E	F
1	A	0.3	0	0.3	0	0	0.4
2	B	0	0.4	0	0.4	0	0.2
3	C	0.3	0	0	0	0.7	0
4	D	0	0.4	0	0	0.3	0
5	E	0	0	0	0	0	1
6	F	0.4	0.2	0	0	0	0.4

Figure 8. Transition Matrix of the development process

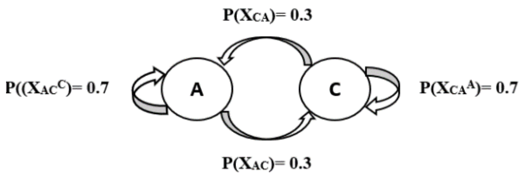


Figure 9. Illustrating iterations between A and C.

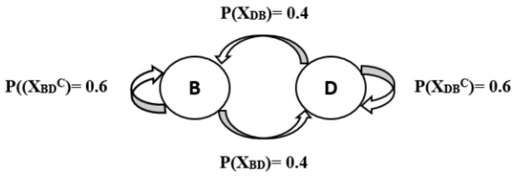


Figure 10. Illustrating iterations between B and D.

The Equation 25 represents the maximum sequential iterations between A and C coupled activities.

$$E\left(x_{(AC)}\right)=2\ \times(1+0.7\times\left(\left(\frac{3\times0.3}{(1\text{-}(0.3)^2}\right)+\left(\frac{2\times0.3}{(1\text{-}(0.3)^2)^2}\right)\right)=2+\ 1.4\ \times\left(\left(\frac{0.9}{0.91}\right)+\left(\frac{0.6}{(0.91)^2}\right)\right)=4.0003\cong4 \qquad (25)$$

The iteration probability values related to the B and D activities is determined according to the Figure 10. Also, the Equation 26 represents the maximum sequential iterations between B and D coupled activities.

$$E\left(x_{(BD)}\right)=2\ \times(1+0.6\times\left(\left(\frac{3\times0.4}{(1\text{-}(0.4)^2}\right)+\left(\frac{2\times0.4}{(1\text{-}(0.4)^2)^2}\right)\right)=2\ +1.2\ \times\left(\left(\frac{1.2}{0.84}\right)+\left(\frac{0.8}{(0.84)^2}\right)\right)=4.789\cong5 \qquad (26)$$

As well as, the iteration probability values related to the (A, F) and (B, F) coupled activities are illustrated according to Figure 11 and 12. The equation 27 represents the maximum sequential iterations between A and F activities.

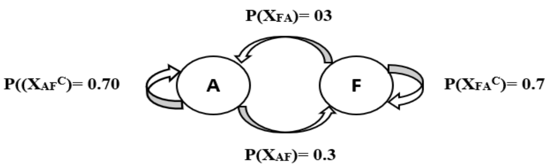


Fig. 11. Illustrating iterations between A and F.

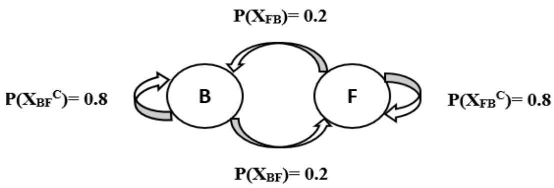


Fig. 12. Illustrating iterations between B and F.

$$E\left(x_{(AF)}\right)=2\ \times(1+0.7\times\left(\left(\frac{3\times0.3}{(1\text{-}(0.3)^2}\right)+\left(\frac{2\times0.3}{(1\text{-}(0.3)^2)^2}\right)\right)=2\ +1.4\ \times\left(\left(\frac{0.9}{0.91}\right)+\left(\frac{0.6}{(0.91)^2}\right)\right)=4.0003\cong4 \qquad (27)$$

The equation 28 represents the maximum sequential iterations between B and F coupled activities.

$$E\left(x_{(BF)}\right)=2\ \times(1+0.8\times\left(\left(\frac{3\times0.2}{(1\text{-}(0.2)^2}\right)+\left(\frac{2\times0.2}{(1\text{-}(0.2)^2)^2}\right)\right)=2\ +1.6\times\left(\left(\frac{0.6}{0.96}\right)+\left(\frac{0.4}{(0.96)^2}\right)\right)=3.694\cong4 \qquad (28)$$

$$M=E\left(x_{(AC)}\right)+E\left(x_{(BD)}\right)+E\left(x_{(AF)}\right)+E\left(x_{(BF)}\right)=4+5+5+4=18$$

Note2- It should be noted that higher transition probability between two coupled activities results in higher length of sequential iterations which logically is reasonable illustrated in the Figure 13. The Equations 29 and 30 shows the sequential iterations between two coupled activities with an example of (p=0.8, q=0.2) and (p=0.9, q=0.1).

$$E\left(x_{(ij)}\ \right)=2\ \times(1+0.2\times\left(\left(\frac{3\times0.8}{(1\text{-}(0.8)^2}\right)+\left(\frac{2\times0.8}{(1\text{-}(0.8)^2)^2}\right)\right)=$$

$$2\ +0.4\ \times\left(\left(\frac{2.4}{0.36}\right)+\left(\frac{1.6}{(0.36)^2}\right)\right)=6.93 \qquad (29)$$

$$E\left(x_{(ij)}\ \right)=2\ \times(1+0.1\times\left(\left(\frac{3\times0.9}{(1\text{-}(0.9)^2}\right)+\left(\frac{2\times0.9}{(1\text{-}(0.9)^2)^2}\right)\right)=$$

$$2\ +0.2\ \times\left(\left(\frac{2.7}{0.19}\right)+\left(\frac{1.8}{(0.19)^2}\right)\right)=14.81 \qquad (30)$$

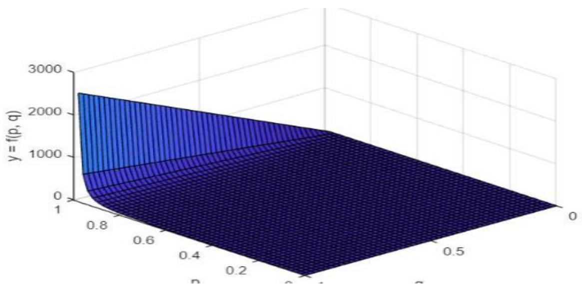


Figure 13. Illustrating the growth of iterations with increasing P.

2-2 Developing the ITM basis vectors for the ILP

The Figure 14 illustrates ITM variables including sequential iterations. The Equation 31 indicates ITM basis vector resulted from the columns in the Figure 14. The Equations of 32 to 39 are the ITM's representative for sequential iterations and the Equation from 40 to 43 represent concurrent iterations between coupled activities.

Sequential Iterations	Coupled Activities	ITM Variables	Sequence Number of Iterations					B
			1	2	3	4	5	
X_S	$X_{(A,C)} = 4$	A1	1	0	1	0		$b_{A_1} = 2$
		C	0	1	0	1		$b_C = 2$
	$X_{(B,D)} = 5$	B1	1	0	1	0	1	$b_{B1} = 2$
		D	0	1	0	1	0	$b_D = 2$
	$X_{(A,F)} = 4$	A2	1	0	1	0		$b_{A_2} = 2$
		F	0	1	0	1		$b_{F1} = 2$
	$X_{(B,F)} = 4$	B2	1	0	1	0		$b_{B2} = 2$
		F	0	1	0	1		$b_{F2} = 2$

Figure 14. Illustrating ITM basis vectors

$$\begin{cases} ITM = (A1, C, B1, D, A2, F1, B2, F2) \quad (31) \\ ITM_{A1} = (1,0,0,0,0,0,0,0) \quad (32) \quad ITM_C = (0,1,0,0,0,0,0,0) \quad (33) \quad ITM_{B1} = (0,0,1,0,0,0,0,0) \quad (34) \\ ITM_D = (0,0,0,1,0,0,0,0) \quad (35) \quad ITM_{A2} = (0,0,0,0,1,0,0,0) \quad (36) \quad ITM_{F1} = (0,0,0,0,0,1,0,0) \quad (37) \\ ITM_{B2} = (0,0,0,0,0,0,1,0) \quad (38) \quad ITM_{F2} = (0,0,0,0,0,0,0,1) \quad (39) \quad ITM_{A1,\,C} = (1,1,0,0,0,0,0,0) \quad (40) \\ ITM_{B1,\,D} = (0,0,1,1,0,0,0,0) \quad (41) \quad ITM_{A2,F1} = (0,0,0,0,1,1,0,0) \quad (42) \quad ITM_{B2,\,F2} = (0,0,0,0,0,0,1,1) \quad (43) \end{cases}$$

X_S, X_C variables represent sequential and concurrent iterations respectively. The Equations 44 and 45 represent the initial Equation of constraints of the ILP model which are substituted by the ITM vectors.

$$\begin{cases} AX \leq B \quad (44) \\ A_s.\, X_s + A_C.\, (X_C + P_R) \leq (B_{A1}, B_C, B_{B1}, B_D, B_{A2}, B_{F1}, B_{B2}, B_{F2}) \quad (45) \end{cases}$$

The Equation 46 calculates reworking of concurrent iterations in which three possible states could be occurred. The Equations of 47 to 50 represent the expected reworking related to the concurrent iterations.

$$\begin{cases} R_{ij} = E\,(X_R) = X_R.\, P_X = 1\times Pr_{ii}.\, (1 - Pr_{jj}) + 1\times Pr_{jj}.\, (1 - Pr_{ii})) + 2\times Pr_{ii}.\, Pr_{jj} \quad (46) \\ R_{(A1,C)C} = 1\times\left(P(x_{A1A1}\right).(1 - \left(P(x_{CC}\right) + 1\times\left(P(x_{CC}\right).(1 - \left(P(x_{A1A1}\right) + 2\times\left(P(x_{A1A1}\right).\left(P(x_{CC}\right) = 0.3 \quad (47) \\ R_{(B1,\,D)} = 0.4 \quad (48) \quad R_{(A2,\,F1)} = 0.18 + 0.28 + 0.24 = 0.70 \quad (49) \quad R_{(B2,\,F2)} = 0.24 + 0.24 + 0.32 = 0.80 \quad (50) \end{cases}$$

The Equation 51 indicates the constraints of the ILP using the ITM basis vectors.

$$\begin{cases} ITM_{A1}.\, X_{A1S} + ITM_C.\, X_{CS} + ITM_{B1}.\, X_{B1S} + ITM_D.\, X_{DS} + ITM_{A2}.\, X_{A2S} + ITM_{F1}.\, X_{F1S} + ITM_{B2}.\, X_{B2S} + ITM_{F2}.\, X_{F2S} + \\ ITM_{A1\,C}.\, X_{A1C} + ITM_{B1,\,C}.\, X_C + ITM_{A2,\,F1}.\, X_C + ITM_{B2,\,F2}.\, X_C + Pr_{(A1,\,C)}.\, X_{A1C} + Pr_{(B1,\,C)}.\, X_{B1C} + Pr_{(A2,\,F1)}.\, X_{A2F1} + \\ Pr_{(B2,\,F2)}.\, X_{B2F2} \leq B \\ (1,0,0,0,0,0,0,0).\, X_{A1} + (0,1,0,0,0,0,0,0).\, X_C + (0,0,1,0,0,0,0,0).\, X_{B1} + (0,0,0,1,0,0,0,0).\, X_D + \\ +(0,0,0,0,1,0,0,0).\, X_{A2} + (0,0,0,0,0,1,0,0).\, X_{F1} + (0,0,0,0,0,0,1,0).\, X_{B2} + (0,0,0,0,0,0,0,1) X_{F2} + \\ +(1,1,0,0,0,0,0,0).\, X_{(A1,C)} + (0,0,1,1,0,0,0,0).\, X_{(B1,C)} + (0,0,0,0,1,1,0,0).\, X_{(A2,F1)} + \\ +(0,0,0,0,0,0,1,1).X_{(B2,F2)} + (1,1,0,0,0,0,0,0).X_{(A1,C)}.R_{(A1,C)} + (0,0,1,1,0,0,0,0).X_{(B1,C)}.\, R_{(B1,C)} + \\ +(0,0,0,0,1,1,0,0).\, X_{(A2,F1)}.R_{(A2,F1)} + (0,0,0,0,0,0,1,1).X_{(B2,F2)}.\, R_{(B2,F2)} \leq (2,\, 2,\, 3,\, 2,2,\, 2,\, 2,2) \quad (51) \end{cases}$$

The Equations 52 to 59 is resulted from the Equation 51.

$$\begin{cases} X_{A1} + 1.3X_{(A1,C)} \leq 2 \quad (52) \quad X_C + 1.3X_{(A1,\,C)} \leq 2 \quad (53) \quad X_{B1} + 1.4X_{(B1,D)} \leq 3 \quad (54) \\ X_D + 1.4X_{(B1,D)} \leq 2 \quad (55) \quad X_{A2} + 1.7X_{(A2,F1)} \leq 2 \quad (56) \quad X_{F1} + 1.7X_{(A2,F1)} \leq 2 \quad (57) \\ X_{B2} + 1.8X_{(B2,F2)} \leq 2 \quad (58) \quad X_{F2} + 1.8X_{(B2,\,F2)} \leq 2 \quad (59) \\ X_{A1}, X_C, X_{B1}, X_D, X_{A2}, X_{F1}, X_{B2}, X_{F2}, X_{(A1,C)},\ X_{(B1,D)},\ X_{(B2,F2)},\ X_{(A2,F1)} = 0,1 \end{cases}$$

The Equations 60 to 67 guarantees completing coupled activities either through sequential or concurrent iterations.

$$\begin{cases} X_{A1} + X_{C1} + X_{(A1,C)} \geq 1 \quad (60) \quad X_{B1} + X_{B1} + X_{(B1,\,D)} \geq 1 \quad (61) \quad X_{A2} + X_{F1} + X_{(A2,F1)} \geq 1 \quad (62) \\ X_{A1} + X_{C1} + X_{(B2,F2)} \geq 1 \quad (63) \quad X_{A1} = X_{C1} \quad (64) \quad X_{A2} = X_{F1} \quad (65) \\ X_{B1} = X_D \quad (66) \quad X_{B2} = X_{F2} \quad (67) \end{cases}$$

3 Developing the objective function

In this section, decision variables and parameters are identified to determine the objective function. There are several parameters and decision variables that should be considered in the ILP model. The below objective function involves the required parameters and variables to optimize the number of sequential and concurrent iterations of the problem.

$$Z=Min \sum_{i=0}^{n} t_{xi_s}.X_{is}+L_{xij_c}.X_{(ij)c} \; + u.(\sum_{i=1}^{n} Rx_{(ij)c}.\,(X_{(ij)c}\,(1+ D_{(ij)_C})\,)) \qquad j=1,2,3,.... \quad (68)$$

X_i- is the number of sequential iterations to complete the ith activity.

X_{ij}- is the number of concurrent iterations.

R_{ij} is explained in the previous section illustrating reworking of concurrent iterations.

t_{xi}- is the required time to complete a sequential iteration.

u- is the coefficient related to the information uncertainty of performing concurrent iterations. ($u \geq 1$)

L_{Xij}-is the required time to complete a concurrent iteration. L is calculated according to the Equation 69. Also, D_{ij} is the time difference two sequential iteration calculated according to the Equation 70.

$L_{ij} = Max\,(t_i, t_j) \quad (69) \qquad D_{ij} = |t_i - t_J| \quad D_{ij} \geq 0 \qquad (70)$

The second part of the objective function indicated by the Equation 71 represents the undesirable element in which the growth of time difference (D_{ij}) would result in increasing rework during concurrent iterations.

$$\sum_{i=1}^{n} Rx_{(ij)c}.\,(X_{(ij)c}\,(1+ D_{(ij)_C})\;) \quad (71)$$

3-1- Defining the input parameters and exploring the results

Step1- the input parameters of the ILP model is illustrated in the Figure 15. It should be considered that L and D parameters are calculated according the 69 and 70 Equations and the reworking parameters (R_{ij}) are estimated in advance.

Sequential iterations				Concurrent iterations			
Variables	Time	Variables	Time	Variables	L	D	R
X_{A1}	3	X_C	7	$X_{(A1,C)}$	7	4	0.3
X_{B1}	4	X_D	5	$X_{(B1,D)}$	5	1	0.4
X_{A2}	3	X_{F1}	6	$X_{(A2,F1)}$	6	3	0.7
X_{B2}	4	X_{F2}	6	$X_{(B2,F2)}$	6	2	0.8

Figure 15. Illustrating the sequential and concurrent iterations parameters

Step2- Establishing the objective Function indicated in the Equation 72.

$Z= Min\;\; 3X_{A1}+4X_{b1}+3X_{A2}+4X_{B2}+7X_C+5X_D+6X_{F1}+6X_{F2}+8.5X_{(A1.C)}+5.8X_{(B1.D)}+8.8\,X_{(A2,.F1)}+8.4X_{(B2.F1)}$ (72)

Step3- solving the ILP using defined objective Function and Constraints in the previous section with U=1.

The Figure 16 illustrates the optimal solution with sequential iterations equal to zero and all the concurrent iterations equal to one.

```
Optimal solution found.
Solution vector:   0   0   0   0   0   0   0   0   1   1   1   1
Optimal value of the objective function = 29
```

Figure 16. Illustrating the optimal solution with u=1

$X_{(A1.C)},X_{(B1.D)},\, X_{(A2,.F1)},X_{(B2.F1)}=1 \quad , \quad X_{A1},X_{b1},X_{A2},X_{B2},X_C,X_D,X_{F1},X_{F2}=0$

Step4- Sensitivity analysis of the ILP model with equal times for sequential iterations and increasing the uncertainty coefficient with U=2.

$Z= Min\;\; 1X_{A1}+1X_{b1}+1X_{A2}+1X_{B2}+1X_C+1X_D+1X_{F1}+1X_{F2}+2.(1.4X_{(A1.C)}+1.7X_{(B1.D)}+1.8\,X_{(A2,.F1)}+8.4X_{(B2.F1)})$

```
Optimal solution found.
Solution vector:     0    0    1    1    0    0    1    1    1    1    0    0
Optimal value of the objective function = 8
```

Figure 17. Illustrating the optimal solution with u=2.

$X_{(A1.C)} = 1, X_{(B1.D)}, X_{(A2,.F1)}, X_{(B2.F1)} = 0$, $X_{A1}, X_{b1} = 0, X_{A2}, X_{B2} = 1, X_{C}, X_{D} = 0, X_{F1}, X_{F2} = 1$

The Figure 17 Illustrates that increasing uncertainty for concurrent iterations results in increasing the number of sequential iterations from zero to 4.

4 Conclusion

This study developed a partitioned model in order to reduce the complexity of coupled activities in the product development process. The first contribution of this study is decomposing of multiple coupled activities through a proposed binary DSM variable named ITM. The ITM was developed in order to establish the basis vectors of equations to solve an ILP optimization model. The ILP model was utilized to optimize the number of sequential and concurrent iterations in different discrete times. The second contribution is that the constraints of the required cycle iteration to accomplish the multiple coupled activities was included in to the ILP model using Markov Chain and ITM. Thus, the model enables designers to cope with coupled activities to reduce a product development process cycle-time to keep up with promised deadline. Especially when there are multiple alternatives of new products, the proposed model can assist product development managers to select those products which fall within project milestones for development.

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