APPLYING ROBUST DESIGN METHODOLOGY TO A QUADROTOR DRONE

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Abstract

Optimization of mechatronics system often rely on optimization of the controller, while treating the structural part of the system as a fixed constraint. The research work presented in this paper has the goal of obtaining a robust design for a quadrotor drone, focusing on the structural parameters of the drone, such as mass and dimensions. A robust design method is a design that focuses on minimizing the effects of the variations of the design parameters, here structural parameters, on the response of the system. In this paper, the system's response is represented by its energy consumption. Using a MonteCarlo simulation, the most influential design parameters are first determined, and then a designer-defined objective function is minimized to obtain a robust mechanical design for the quadrotor at hand. The optimized drone is then shown consuming less energy than a comparable drone, while also being more robust to variation of design parameters.

Keywords: Case study, Mechatronics, Robust design, Optimisation

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1 INTRODUCTION

Design of mechatronic systems is inherently complex due to their multi-domain nature. This different engineering domains, involved in the design activity, influence each-other during the design process, making the design a tedious task to achieve for design engineers (Mohebbi et al., 2014). Traditionally, a mechatronic system is designed sequentially, the mechanical part first and then the electronic components followed by the control strategy. To obtain a more optimized design, the coupling between the different components and domains must be evaluated in early stages of the design process in order to avoid negative dependencies effects (Alyaqout et al., 2011). Many have suggested different methods with the goal of obtaining a better design which incorporate both the mechanical and control aspects of the mechatronic system. The methods proposed tend to focus on optimizing one aspect of the system, for example, the control, or the mechanisms in a disconnected way.

This paper presents a detailed case study of a quadrotor drone robust design. The quadrotor drone is an interesting case study for this research work mainly because it is a complex mechatronic system, involving various domains such as aerodynamics, structure, control, and electronics. Additionally, quadrotor drones are increasingly present in today's world and are used for many different applications; however, they are still costly, even the ones used for entertainment purposes. Small to medium drones are often powered by batteries, which are always a concern in autonomous mechatronic systems due to their relative high weight and to the fact that they limit the maximum autonomy of the system.

Some methods were proposed in the literature for the design support of drones or mechatronic systems in general. One of such methods is Design for Control (DFC) strategy applied to visually served drones presented by Mohebbi et al. (2015). DFC involves the simplification of the dynamic modelling of the system, to fully understand it and ease its representation, and then designing a control algorithm which improves the control of the system. Another design method is the robust structure-control design, which proposes the use of a non-linear dynamic multi-objective optimization to design a system which considers the interaction between the structure and the control to propose a robust design as presented by Alyaqout et al. (2011). In this method, the robustness of the system is mostly achieved by the design of the controller only, limiting it practically to a robust control approach. In other words, in both these methods it is the control part that is the design focus and no major information is obtained about the mechanisms; additionally, the interaction goes in a single direction from control to mechanism while the other way is only achieved by further simplifications of the dynamics by adding extra constrains such as stability criterions (Mohebbi et al, 2015).

This paper will present the robust design of a drone, especially of its structure. Applying an optimization method to obtain a robust design of a quadrotor drone could eventually lead to a better evaluation of the parameters variation on the quadrotor energy consumption and hence resulting in quadrotor drones with lower energy consumption, increased battery life and lower cost.

The basics of robustness and the concept of a robust design were notably introduced by Taguchi and Wu (1979). A robust design was then defined as a design that is insensitive to noise or small variations in the system's parameters or inputs. Robust design methods are employed in many different domains and applications. Choi et al. (2008), designed, on a molecular level, robust materials. Zang et al. (2005) identified that one of the most successful field of application for robust design is mechanical engineering design, especially in static performance.

The design method explored in this paper focuses on designing a robust mechanical aspect of the mechatronic system while in parallel always considering its control strategy. Both the mechanism and the control strategy are considered as wholes and are not further simplified to ease the optimization process like it is done in DFC. As said before, to realize a robust design is to achieve a better control over the effect of these variations by designing the most resistant system to uncertainties and variations in design parameters. The variations and uncertainties in design parameters could generally result from the fabrication methods, the fabrication process as reported in Caro et al. (2005). Abraham et al. (2001), proposed two ways of reducing the variations effects. First, they proposed to control the noise, which is not always easy or even feasible. Second, they suggested exploiting the interaction between noise, variation and their effects. Chen and al. (1996), defined two categories of problem, the first Type I where the system is optimized by "minimizing the variations in performances caused by variations in noise factors (uncontrollable parameters)" and Type II where the system is optimized by "minimizing variations in performance caused by variations in control factors (design variables)". Allen et al. (2006), added a third type, Type III, where the system is optimized to minimize the effects of the uncertainty of the system model. In this paper, we will use a Type II optimization.

We rely on a MonteCarlo based simulation to induce uncertainties in design variables and then analyze their effect on the quadrotor drone responses. Design variables are parameters of the system (i.e. physical parameters: mass, components dimensions) that are chosen by the designer and the responses are different responses of the mechatronic system (i.e. rise time, energy consumption, etc.). A designer-selected design objective function, which involves the results of the MonteCarlo simulation, is then optimized using boundaries, equalities and inequalities to minimize the defined function. The response studied here will be the energy consumption. As mentioned above, drones are powered by batteries that limit the maximum flight time of the system. A robust drone in terms of energy consumption would mean that this drone consumes less energy that an initial design, for the same task, and those variations on the drone's physical parameters would also have less effect of the energy consumption. Therefore, achieving a robust design can lead to a better understanding of the variations of the parameters of a quadrotor drone, which in turn can yield new designs with looser tolerances and eventually lower production costs. As identified by Hasenkamp et al. (2008), most of the models proposed in the literature are either pedagogical or academic takes on robust design and would be hard to implement in industry. The methodology presented here could easily be applied by any drone designer to analyze and design a drone for robustness.

It is worth noting that we also tested the effect of a robust design approach on the time response, the time response being a major performance variable in the time domain for dynamical systems, however, the results showed that the time response was mainly affected by the controller gains than the physical parameters of the drone itself. Therefore, this was left out of this paper.

2 SYSTEM MODELING AND PARAMETER ANALYSIS

The model used in this paper, for the quadrotor drone dynamics, controls the system's position in terms of the altitude z and the attitude roll, pitch, yaw (ϕ, θ, ψ), as defined in Equation 1. These represent the state variables of the quadrotor drone as represented by the tensor X:

$$X = [z(t) \quad \phi(t) \quad \theta(t) \quad \psi(t)] \tag{1}$$

The quadrotor dynamic equations are used as developed by Bouabdallah and Siegwart (2007b), no further simplification of the dynamics or extra stability constraints will be applied. Variables presented in Equation 1 are function of time (t), but for clarity will not be noted as such. The equation of movement can be summarized by the following important ones:

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}a_1 + \dot{\theta}a_2\Omega_r + \frac{U_2}{I_{xx}} \\ \ddot{\theta} = \dot{\phi}\dot{\psi}a_3 - \dot{\phi}a_4\Omega_r + \frac{U_3}{I_{yy}} \\ \ddot{\psi} = \dot{\theta}\dot{\phi}a_5 + \frac{U_4}{I_{zz}} \\ \ddot{x} = \frac{u_x U_1}{m} \\ \ddot{y} = \frac{u_y U_1}{m} \\ \ddot{z} = g - \frac{(\cos\phi\cos\theta)U_1}{m} \end{cases} \tag{2}$$

where the $a_i, i \in [1,5]$ and $b_k, k \in [1,3]$ are physical constants of the quadrotor defined as:

$$\begin{cases} a_1 = \frac{(I_{yy} - I_{zz})}{I_{xx}} \\ a_2 = \frac{J_r}{I_{xx}} \\ a_3 = \frac{(I_{zz} - I_{xx})}{I_{yy}} \\ a_4 = \frac{J_r}{I_{yy}} \\ a_5 = \frac{I_{xx} - I_{yy}}{I_{zz}} \end{cases} \tag{3}$$

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = b(-\Omega_2^2 + \Omega_4^2) \\ U_3 = b(\Omega_1^2 - \Omega_3^2) \\ U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \\ \Omega_r = \Omega_1 + \Omega_3 - \Omega_2 - \Omega_4 \\ u_x = \cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi \\ u_y = \cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi \end{cases} \quad (4)$$

Where $\phi\dot{\phi}\ddot{\phi}, \theta\dot{\theta}\ddot{\theta}, \psi\dot{\psi}\ddot{\psi}$ are the Euler angles and their associated speeds and accelerations, $\ddot{x}\dot{x}x, \ddot{y}\dot{y}y, \ddot{z}\dot{z}z$ are the Cartesian accelerations, speeds and positions in the world-fixed frame, I_{xx}, I_{yy}, I_{zz} are moments of inertia around x, y, z axis respectively, J_r is the rotor inertia, l is the propeller distance to the center of gravity (CoG) of the quadrotor and m is the mass of the quadrotor. Also Ω_i with $1 \leq i \leq 4$ are the propeller angular rates, and Ω_r is the overall residual propeller angular rate, b and d are respectively thrust and drag factor determined experimentally. Finally, U_i are the inputs of the system defined by Equation 4. The main variables are also illustrated in Figure 1.
It is important to note that these equations of motion hold only for small variation of the Euler angles ϕ, θ, ψ around the drone's equilibrium (null angles). This model was chosen for its simplicity and for the fact that the energy consumption of the drone in response to step command is studied. It would be interesting to eventually compare the results obtained with this model to results obtained with a complete model of the drone and for more athletic manoeuvre.

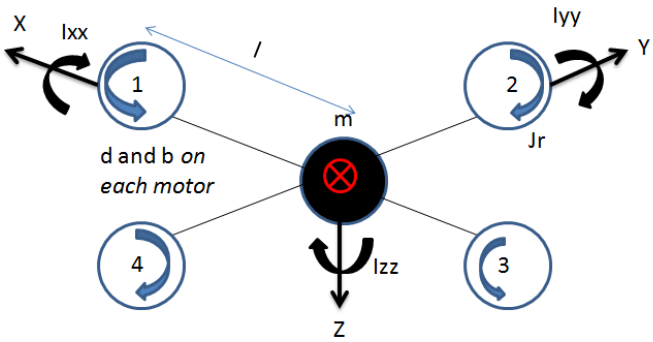

Figure 1. Quadrotor model with its design parameters

The system is modelled in Simulink, using 4 Proportional Derivate (PD) controllers, sending a reference signal, one for each state variable, defined in Equation 1.
Using this closed loop system, it is now possible to analyze the effect of each physical variable of the quadrotor and the effect of the uncertainties associated to them on the system's response. The physical variables are those that would normally be chosen during the structural design of a system. These are generally referred to as non-real-time parameters (NRTP), meaning that they are normally set in time and are hard to change when the design is fixed as explained by Mohebbi et al. (2015).
The robust design method presented here deals with NRTP, while the controller is set to a PD, it can however be refined using a dedicated method such as DFC, or a traditional poles placement method. A vector p containing all of the NRTPs considered here is defined as follows:

$$p = [I_{xx} \quad I_{yy} \quad I_{zz} \quad J_r \quad b \quad d \quad l \quad m\,] \quad (5)$$

The effect of the uncertainties on the response for each parameter, evaluated through the energy consumption, need therefore to be calculated. This is accomplished using a MonteCarlo simulation, which is a statistical process that repeats multiple times a process using a random sampling of data. The MonteCarlo simulation loop analyses the impact of each parameter variation on the energy consumption (Egel, 2009). The simulation runs the model of the quadrotor drone N number of times, generally more than a thousand times (Mooney, 1997). From this we obtain a distribution of energy consumption values (explained further in the paper), which in turn can be analyzed to obtain results on the effect of each parameter.

First, each element of the vector p is affected by uncertainties, either due to experimental measurements of the parameter, or from the manufacturing process. The uncertainties can be represented as forming a normal distribution, having a specific mean ($\mu_{I_{xx}}$) and variance ($\sigma_{I_{xx}}^2$) (Du et al., 2009):

$$Ixx \in N\left(\mu_{I_{xx}}, \sigma_{I_{xx}}{}^2\right) \quad (6)$$

Firstly, each parameter is selected randomly N times from the normal distribution. Secondly, a response function is defined. In this paper, the function is the electrical energy consumed by the quadrotor drone which is proportional to the squared speed of the propellers. It is, therefore, possible to summarize it as the following expression:

$$e(t) \propto \Sigma\Omega(\mathrm{t})^2 \propto U_1 \quad (7)$$

where $e(t)$ is the energy consumed as a function of time, $\Omega(t)$ the speed of the propellers as a function of time and U_1 the input defined in Equation 4.

A value proportional to the energy consumed by the quadrotor can be directly extracted from the mathematical model using Equation 7. Using the previously defined vectors of parameters and the response function, the MonteCarlo simulation evaluates the energy consumed by the quadrotor for a step excitation input for each of the system state variables.

The energy consumption is then integrated over time to obtain the total energy consumed by the quadrotor for each simulation of the MonteCarlo analysis. A histogram of the distribution of the total, integrated energy consumption shows that it approximates a normal distribution, when using a normal distribution to evaluate the uncertainties on the NRTPs as illustrated in Figure 2.

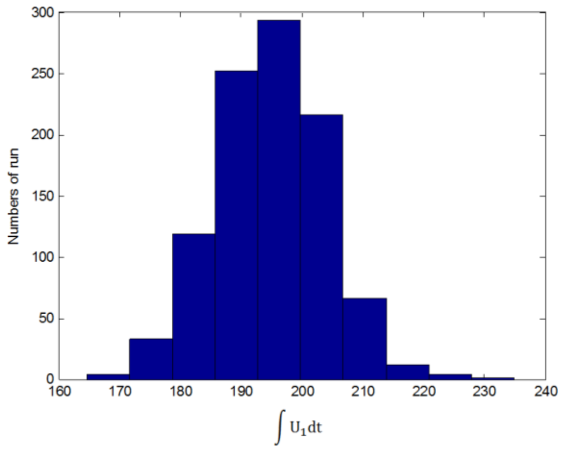

Figure 2. Distribution of the energy consumption obtained by Monte Carlo simulation for a quadrotor drone

Using the newly obtained results, it is now possible to analyze which parameters of the p vector (Equation 5) have the most important effect on the consumed energy.

To this purpose, a linear curve is fitted through each parameter with respect to the energy consumed to determine the most effective parameters. The slope of each regression is then multiplied to the difference between the minimum and maximum value of each parameter vector to get an absolute value of the energy consumption induced by the variation in the parameters. The y-axis value on the Pareto plot (Figure 3) can be determined using the following equation:

$$\Delta E = slope * (\max(p) - \min(p)) \quad (8)$$

where E is the total energy used by the quadrotor and p is the value of the parameter analyzed.

Analyzing the results illustrated in Figure 3, it is easy to determine that the mass of the quadrotor is the most influencing parameter on the energy consumption. Knowing which parameters are the most important and how the variation of these parameters affect the response of the quadrotor, it is possible for the design engineer to develop a better strategy to improve the design of the quadrotor. This will be carried out in Section 3.

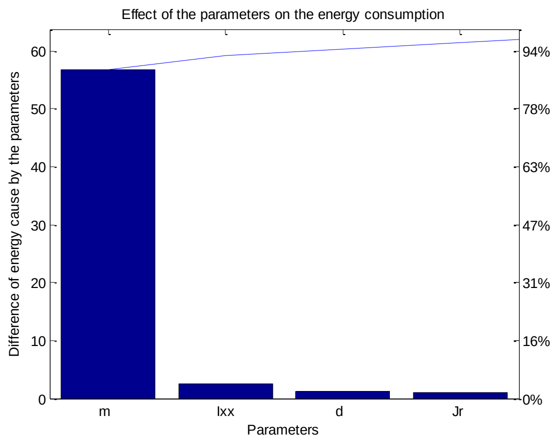

Figure 3. Pareto plot of the most influential parameters obtained by Monte Carlo analysis of the energy consumption of a quadrotor drone

3 QUADROTOR DRONE ROBUST DESIGN METHODOLOGY

The MonteCarlo simulation analysis is now integrated into a robust design optimization double-loop.
A design variable vector is first determined from the quadrotor model. The same vector p from Equation 5 is used here. These are the variables that the method optimizes to obtain a robust design. The goal of the method is to minimize an objective function defined by the design engineer. Here we use the MATLAB function *fmincon* for the optimization loop. As mentioned in Zang et al. (2005) review on robust design, stochastic heuristic methods such as particle swarm optimisation and evolutionary algorithms seem to be a good alternative for this type of problem in comparison with standard optimisation algorithms such as the one used in this paper.
To obtain a robust design, the function must at least be composed of the standard deviation of the response, here the energy consumption, obtained by the MonteCarlo simulation. The optimization algorithm then finds the value for each parameter of the vector p which will minimize the objective function f as described by Equation 9.

$$f = \omega_1 \mu + \omega_2 \sigma^2 \tag{9}$$

where ω_1, ω_2 are weight factors determined by the designer, μ is the mean of the energy consumed and σ is the standard deviation of the distribution of the energy consumed. The sample mean and the sample variances are calculated from the results of the MonteCarlo simulation as follows and such as defined in Park et al. (2006):

$$\begin{cases} \mu = \frac{1}{\mathrm{N}} \sum_{i=1}^{N} E_i \\ \sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (E_i - \mu)^2 \end{cases} \tag{10}$$

where E is the energy consumed by the quadrotor during simulation run number i where $i = [1, N]$.
The minimization procedure can take in different optimization constraints, such as inequalities or equalities between the parameters, or between a parameter and a constant. The optimization algorithm starts from a starting point x_0 and can also use lower and upper boundary vectors for the parameter, such as those used in this case study.
These values are mostly determined by validating if the controller still applies correctly to the system using those upper and lower boundaries. The algorithm will then run the double-loop MonteCarlo simulation numerous times, until it finds a minimum in the objective function defined by equation 9.
The applied method can be summarized as follows:

1. The designer chooses an objective function to be minimized (such as Equation 9). The lower and upper boundaries vectors as well as the initial values are to be defined.
2. Using the initial values of the mean for each parameter distribution, a vector of dimension N is generated for each parameter defined in Equation 5.
3. For each value of the parameters, the response (energy consumption) is calculated. This is the MonteCarlo simulation which is repeated N times. This step is the first loop of the algorithm.

4. Using the results of the MonteCarlo simulation, the standard deviation and mean of the simulation are calculated.
5. The optimization algorithm generates a new set of values for the mean of each parameter and returns to step 2. This is the second loop of the algorithm.
6. When the minimum of the objective function is obtained, the value of each parameter is found.

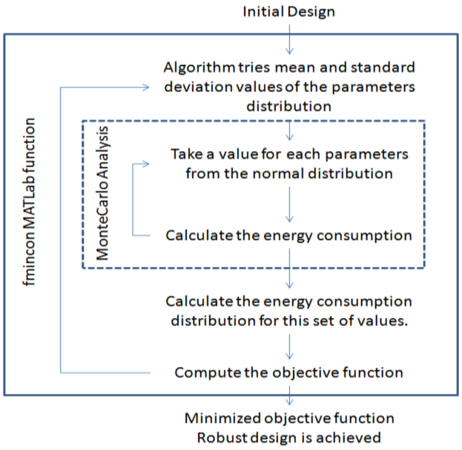

Figure 4. Robust design algorithm used for quadrotor design

Figure 4 presents an illustration of the double MonteCarlo-based algorithm which helps design the mechanical part of the system while considering the chosen control strategy (the applicability is constantly evaluated). It is worth noting that in the first loop the MonteCarlo simulation calculates the distribution of the response for a set of values, and then in a second loop, the optimization algorithm sets varied values of the parameters vector along the chosen distribution in order to minimize the objective function (minimizing energy consumption).

4 APPLICATION TO A QUADROTOR DESIGN AND PARAMETER ANALYSIS

The quadrotor is first modelled in Simulink using the equation of motion (Equations 1 to 3) and 4 PD controllers. The initial set of values will be the initial vector (x0) for the optimization algorithm. The PD controller gains are tuned so that the time responses of the quadrotor (in both altitude and attitude) are as desired. In Section 2, it was concluded that the mass was the most influential parameter on the quadrotor design such as presented in Figure 3. Analyzing the other parameters effects, it is clear, to a lesser degree, that the moments of inertia are also affecting the response. Since the moments of inertia are in fact mostly based on the mass of the quadrotor, the moments are approximated by Equations 11. These equations assume that the quadrotor center is a sphere of mass M and radius R, and that each motor is at a length l of the CoG, and have a mass of m.

$$\begin{cases} I_{xx} = I_{yy} = \frac{2MR^2}{5} + 2l^2 m \\ I_{zz} = 2\, I_{xx} \end{cases} \tag{11}$$

With those approximations, a lower boundary is defined for the moments of inertia in x and y directions as being the inertia caused by the motor's mass. A lower boundary is also defined for the inertia in z direction. These inequalities are defined in Equation 12.

$$\begin{cases} (I_{xx} = I_{yy}) \geq 2l^2 m \\ I_{zz} \geq 2I_{xx} \end{cases} \tag{12}$$

It is harder to directly set the values of the parameters J_r, d and b, in comparison to the inertia moments or the mass, but they are still treated as design parameters to evaluate their variations effect on the quadrotor. Finally, the mass (m) and arm's length (l) parameters' boundaries are fixed following specifications on the size of the drone. Here we wanted to have a relatively small drone, so the mass and arm's length are fixed accordingly.

Therefore, the lower and upper boundaries for the design method are determined. This will assure the results to be within those boundaries and that the designed quadrotor will still fulfil the specifications and will be controllable by the chosen PD. The values for each vector (x_0, lower and upper boundaries) are detailed in Table 1. The mean and variance of each parameter are listed in Table 1. The nominal values, also called initial values, used in this paper are taken as is from Bouabdallah's thesis (2007a). The standard deviation is then approximated to a value of about 10% of the mean, for simplicity purposes. However, this value can easily be altered by the design engineer to reflect the real distributions of the parameters.

Table 1. Numerical values of the parameters

	x_0	σ	*Lower*	*Upper*	*Optimized*
I_{xx}	7.5e-3	7.5e-4	5e-3	1e-2	1e-2
I_{yy}	7.5e-3	7.5e-4	5e-3	1e-2	1e-2
I_{zz}	1.3e-2	1.3e-3	7e-3	5e-2	5e-2
J_r	6.5e-5	6.5e-6	5e-5	1e-4	9.68e-3
d	3.13e-5	3.5e-6	3e-5	4e-5	3e-5
b	7.5e-7	7.5e-8	7e-7	8e-7	8e-7
l	0.23	0.01	0.15	0.45	0.45
m	0.65	0.05	0.55	1	0.5564

The developed algorithm returns the optimized solution for the p vector. This vector corresponds to a quadrotor model where the objective function is minimized and therefore its performance is less sensitive to the variations in the parameters' values.

We observe that the values of the moment of inertia and the arm's length are equal to those of their upper boundaries. The most optimal design for a quadrotor drone is therefore probably with values higher than those selected, but considering the specifications of our design (defined by the designer) they are the optimal values in this case.

The performances of the optimized design, obtained by the presented method are compared to those of the initial design as proposed by Bouabdallah's thesis (2007a) (see Table 2). Figure 5 presents the histogram of the distribution of the energy consumption for both the optimized design and the initial design.

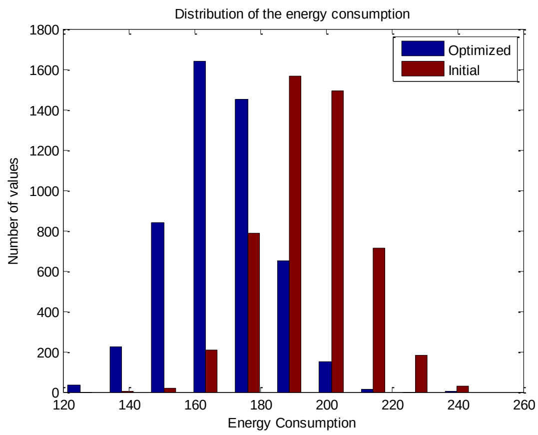

Figure 5. Comparison between the energy consumption for both the optimized and initial design of a quadrotor drone

The numerical values of the distributions are compared in Table 2, where it is shown that the optimized design indeed obtains better results for the mean and variance of the energy consumption.

Table 2. Distribution parameters comparison or the optimized and initial design of a quadrotor drone

Distribution parameter	μ	σ	*Objective function*
Optimized design	164	194	179
Initial design	217	231	224

Figure 5 and Table 2 shows together that the optimized quadrotor drone design obtains a lower mean and a smaller standard deviation than the initial "functional" design. This also means that the quadrotor consumes less energy and is less affected by the variations of its parameters and therefore is more robust. The lower energy consumption can have a large positive effect on the size of the batteries. Indeed, lower energy consumption can lead to the use of smaller batteries or, for the same batteries, to a longer flight time before recharge. This can help us design smaller and more compact quadrotors or quadrotors with longer autonomy.
The method applied here to a quadrotor design case can also be modified to be applied to any mechatronic system. It is worth noting, that we used the energy consumption as the optimization criterion, but any other performance metric could have been chosen by the design engineer or even a combination of several ones.

5 CONCLUSION

In this paper, an application of a standard robust design approach to a quadrotor drone was presented. The results showed that the optimized design obtained was more robust to variations than the initial "functional" design selected from literature. The effects were noticed both on the mean of the energy consumption distribution and the variance of this distribution. The mean of the energy consumption distribution was reduced by 25%. Technically, the mean represents the sum of the squared rotor speeds, but since it is proportional to the energy consumption, we can assume that the 25% drop is also obtained for the energy consumption.
The robust design methodology used in this paper could also be applied to support the design of any mechatronic system in order to achieve more robust designs and allow designers to have a better control over the effect of uncertainties on each designs parameter of the system. The used method is based on a double-loop MonteCarlo simulation to analyze the effects of the design uncertainties and mainly optimize non-real time parameters while always considering the real-time parameters in the design loop. The controller used was a standard proportional derivative; however, future work will include the full design synthesis of the controller within the optimization loop using gains scheduling.
To go further on this project, different optimisation algorithm and heuristics will be compared. Evolutionary algorithm and particle swarm optimisation seems to be promising candidate. Different weights and objective functions will also be tested to see their impacts on an optimal design of the drone.

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