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Tolerance analysis of mechanism taking into account the interactions between deviations using meta-models

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Abstract
A product's functionality depends largely on the interaction of its components and their geometries. Hence, tolerance analyses are used to determine the effects of deviations on a functional key characteristic – also for mechanism. However, possible interactions between the different deviations and the resulting effects among themselves as well as on the functional key characteristics are not considered yet.
This paper considers the extension of an existing "integrated tolerance analysis of systems in motion" approach in detail: The integration of meta-models, which represent the interactions, into the system's functional relationship. Therefore different meta-modeling techniques as well as validation techniques are used. Consequently a recommendation can be derived, how the product developer can achieve a high prediction quality of the meta-models and therefore a reliable tolerance analysis of mechanism taking into account the appearing interactions.

Keywords: tolerance analysis, mechanism, meta-modeling, data-mining, cross-validation.

1 Introduction
The success of a product‘s development is essentially affected by its functionality. Hence, a successful and economical product development goes hand in hand with the ambition to ensure the products functionality as early as possible in the product development process. The product‘s functionality depends largely on the interactions of its components which are influenced by geometrical deviations from their nominal geometries.
Also operation-depending deviations (like deformation due to inertia forces) [1] and therefore varying operating parameters (e. g. temperature) as well as additional varying parameters (e. g. density) are affecting the product's functionality. Since manufacturing-caused deviations appear earlier during the product life-cycle, they also affect the operation- depending deviations which first appear during the product's use. Consequently interactions among deviations and varying parameters affect the product's functionality as well.
In today's product development usually a tolerance analysis is used to determine how the appearing deviations are affecting a system's functional key characteristics. Interactions between these appearing deviations, the correlations with additional varying parameters as well as the effects on a functional key characteristic are not considered yet.
This paper focuses on the extension of an existing approach – the "integrated tolerance analysis of systems in motion [2]" – in detail. As a result the product developer will be able to get information about the effects of deviations and varying parameters on a functional key characteristic, as well as the effects among themselves (due to interactions). In order to take into account the interactions, appropriate mathematical formulations (meta-models) are

needed. However, the prediction quality of an already considered meta-modeling technique – the response surface methodology – isn't sufficient in general [3]. Consequently, additional meta-modeling techniques will be investigated concerning their prediction quality to achieve a more efficient and reliable "integrated tolerance analysis of systems in motion".

2 State of the art

2.1 Tolerance analysis of mechanism

The large diversity of analyzed products and the resulting requirements towards the tolerance analysis and tolerance synthesis reflect in existing and current research activities dealing with many different aspects concerning products and processes. However, existing approaches of the tolerance analysis aren't integrating the specific aspects of systems in motion.

The kinematic behavior of a mechanism is essentially affected by geometric deviations of its components. These deviations can be traced back to e. g. manufacturing discrepancies. So a time-depending tolerance analysis of the effects of geometrical deviations is necessary. Several publications are considering these manufacturing deviations for mechanism with both lower [4] and higher kinematic pairs [5].

Aside geometric deviations due to manufacturing discrepancies also operation-depending deviations are appearing. The operation-depending displacement of components due to joint clearance is considered in [6], while [7] takes into account the appearing deformation of components due to the forces resulting from the system's motion.

[8] considers manufacturing-deviations as well as operation-depending deviations. However, the time-dependence of the mechanism and the deviations remain unconsidered. A robust design approach is presented in [9], which enables the product developer to analyze a system in motion with manufacturing deviations and imperfect joints. Furthermore [2] present the "integrated tolerance analysis of systems in motion" allowing the time-depending tolerance analysis of mechanism with both manufacturing-caused and operation-depending deviations (deformation and displacement due to joint clearance). Moreover, an appropriate visualization of the results of the "integrated tolerance analysis" is shown in [2].

However, all listed research just focuses on the effects of manufacturing and/or operation-depending deviations on defined functional key characteristics of a technical system. The aspect of possible interactions between the different deviations and the resulting effects among themselves as well as on the functional key characteristics are not considered yet.

2.2 Meta-modeling in tolerance management / robust design

The meta-modeling techniques are widely used in current product development. With the possibility to predict a systems response for any input parameter constellation several benefits could be drawn: e. g. time-and money-consuming simulation runs of additional constellations aren't necessary, using appropriate optimization techniques and algorithms a product's optimal (e. g. robust) design can be identified. However, their appearance in the context of tolerance management / robust design is currently still limited.

Meta-modeling techniques are used in tolerance management / robust design mainly in two ways: On the one hand, the functional relation between the varying input parameters and the functional key characteristics is described [10]. On the other hand, the manufacturing costs and the dependencies of varying input parameters and their corresponding tolerances are formulated using meta-models. Especially in case of tolerance allocation, the cost-tolerance function is usually unknown and therefore approximated using usually the response surface methodology [11]. However, also additional meta-models can be adapted to tolerance-related approximation problems.

3 Work methodology

This paper considers a main modification of the existing "integrated tolerance analysis of systems in motion" in detail. Therefore the same demonstrator – a crank mechanism inside a four-stroke, single-cylinder combustion engine – as well as the interactions of appearing deviations, presented in [3], are used.
The extension includes the modification of the mathematical relations between the product's characteristics as well as the appearing deviations and the functional key characteristics (section 4). By integrating an appropriate meta-model (e. g. response surface) - representing the operation-depending deviations - into this relation also the interactions can be taken into account. Although, the used response surface (2^{nd} order) works quite well for some kinds of deviations, its prediction quality is not sufficient for other deviations.
Consequently, the data set has to be analyzed in terms of significant potential for prediction quality improvement (section 5.1). Based on the optimized data set the response surface methodology as well as three additional meta-modeling techniques (artificial neural networks, ANOVA-based support vector regression and W5-decision trees) will be used to describe the interactions (section 5.2). Therefore, in section 5.3, two different sets of investigations are considered, based on the two different chosen validation methods: Split-Validation and Cross-Validation [12]. Hence, the most efficient and reliable meta-modeling technique can be identified. Moreover, the influence of the chosen validation method on the prediction quality can be detailed. In conclusion a recommendation can be derived, how the product developer can achieve a high prediction quality of the needed meta-models and therefore a reliable tolerance analysis of mechanism taking into account the appearing interactions (section 6).

4 Extension of the existing "tolerance analysis of systems in motion"

The "integrated tolerance analysis of systems in motion" consists of three main steps. At first, the mathematical relations between the system's varying (input) parameters and the defined output parameters (functional key characteristics) are needed. Therefore usually vector chains are set up, which include the system's characteristics, the varying parameters as well as the appearing deviations. The second step focuses on the selection as well as on the application of a tolerance analysis method in order to determine the output parameters for a defined number of samples of the system in motion. Therefore the existing approach uses a Monte-Carlo-Simulation. The final step includes the representation and the interpretation of the results [2].
The extension of the existing approach towards considering interactions requires two main modifications, detailed in figure 1.

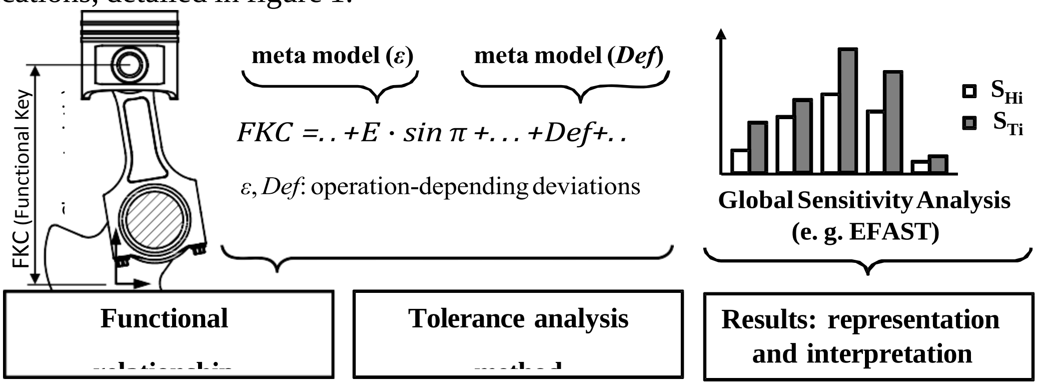

Figure 1 Modifications of the integrated tolerance analysis of systems in motion [3]

On the one hand, the interactions need to be considered when formulating the functional relationship during the first step of the tolerance analysis. Therefore the vectors of the relation's vector chain, which are representing the operation-depending deviations, are replaced by appropriate approximations (e. g. response surfaces). On the other hand, the

interactions as well as their effects on the functional key characteristics must be identified for the representation and interpretation of the results. Therefore the common contributor analysis (e. g. High-Low-Median) is replaced by a global sensitivity analysis (in this case: EFAST).
In order to show the modified approach's practical use, a tolerance analysis of a non-ideal crank mechanism was performed in [3]. The appearing deviations are affecting the precise motion of the crank mechanism's components. These deviations can cause collisions of the components among themselves (e. g. piston / crank shaft) or collisions with additional engine parts (e. g. piston / valves). The crank shaft rotates with a speed of 3000 min^{-1} resulting from the time-depending piston pressure with a maximum of 180 bar [3]. A motion sequence of the crank mechanism consists of two crank rotations with a total crank angle of φ = 720°. The crank mechanism and the appearing manufacturing-caused deviations are shown in figure 2a. Aside of varying operation parameters (viscosity and temperature of the engine oil, young's modulus and density of crank shaft) additional deviations appear:

- Manufacturing deviations (crank radius r, con rod length l, position of the piston e)
- Operation-depending deviations (deformation of the crank shaft, displacement due to joint clearance s)

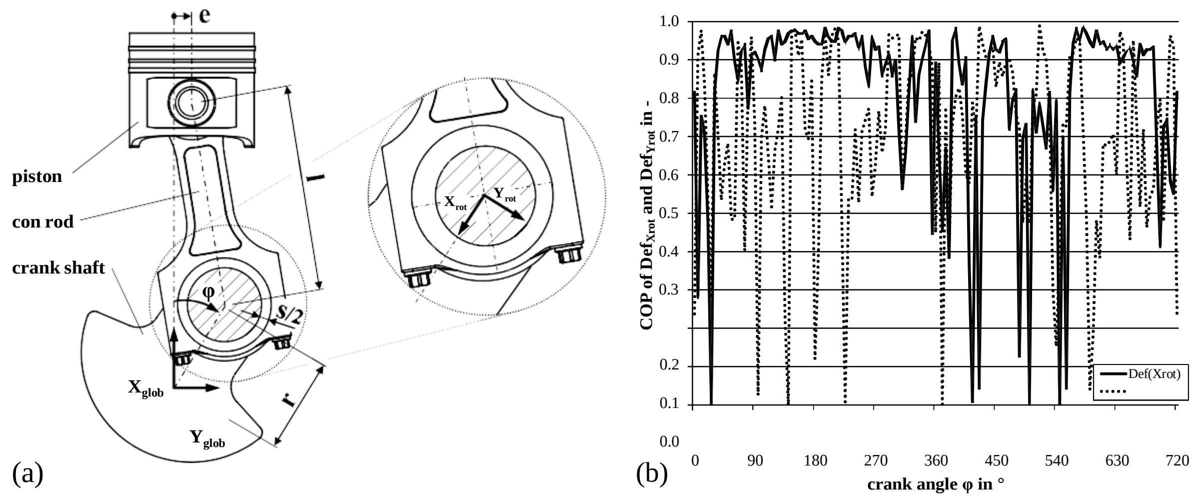

Figure 2 (a) Crank mechanism (b) Time-depending COPs of the crank deformation

In order to consider the appearing interactions during a tolerance analysis, the effects of varying parameters on operation-depending deviations were investigated and approximated by corresponding meta-models using the response surface methodology. However, the prediction quality (criteria: coefficient of prognosis COP [13]) of these meta-models is not sufficient in general. The used response surfaces (2^{nd} order) work quite well for the effects on the operation-depending displacement (high COPs) [3]. However, the prediction qualities of the meta-models concerning the crank shaft's deformation are not sufficient. The time-depending COPs of the deformation's components Def_{Xrot} and Def_{Yrot} are shown in figure 2b.

5 Meta-modeling of the operation-depending deviations

According to figure 2b, it is essentially to improve the prediction quality of the used meta-models (of the crank shaft's deformation) to ensure a reliable tolerance analysis. Therefore, two possibilities will be taken into account: The optimization of the data set for the response surface methodology as well as the use of additional meta-modeling techniques.

5.1 Optimization of the data set

The prediction quality of a response surface is essentially affected by the accuracy of the data used for the generation/training of the meta-model. Since the deformation of the crank shaft has been determined using multi-body-simulations as well as additional finite-elementanalyses [3], a rerun of these simulations with a higher accuracy (valid digits of the values) is done. Furthermore, a normalization of the data set's values is used. Therefore the range of each input parameter x_i and output parameter y_i (deformation) of the meta-model is transformed to the interval [0; 1], where the value 0 means the lower specification limit and the value 1 corresponds to the upper specification limit of each parameter.

5.2 Meta-modeling techniques

Based on the optimized data set the response surface methodology as well as three additional meta-modeling techniques (artificial neural networks, ANOVA-based support vector regression and W5-decision trees) will be used to determine appropriate meta-models for the deformation's components Def_{Xrot} and Def_{Yrot}.

5.2.1 Response surface methodology (RSM)

In order to formulate the relations between the input parameters x_i and the two parameters of the operation-depending deformation y the RSM with a quadratic approximation (β_i: regression coefficients) is used for each point in time of the motion sequence [14]:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum \sum_{i<j=2}^{k} \beta_{ij} x_i x_j + \vartheta$$

However, there still remains an approximation error ϑ. Therefore additional samples (taken from the oversampling) are used to determine this error based on the method of least squares.

5.2.2 Artificial neural networks (ANN)

An artificial neural network contains several artificial neurons and connections among them. Each neuron receives an input-vector and delivers a certain output, depending on the weight w_i and a transfer function. Usually this function is non-linear. The neural network is trained with a data set containing input values as well as output values. After the training the network can calculate the correspondent output y to a specified combination of input-values x_i [15].

$$y = r\left(\sum_i w_i \cdot x_i\right)$$

5.2.3 W5-decision trees (M5)

Decision trees are a simple meta-modeling technique for straightforward problems. However, there is a large, but finite number of possible tree solutions for a certain problem [16]. The model must be both accurate and fast. Therefore several algorithms have been developed, e. g. the so-called M5 [17]. In this model a conventional decision tree is combined with linear regression functions at the leaves of the tree without compromising the accuracy [17].

5.2.4 Support vector regression (SVR)

The approach for support vector regression starts with the nonlinear mapping of the support and test vector into a feature space of higher dimension than the vector itself. Subsequently the dot product of each support vector with the test vector is calculated and summed up, while taking the weight of each vector into account [18]. In the following calculations an ANOVA Kernel is used for the expression *K(x,y)*, since these kernels provide far better results [19].

$$a\left(\sum v_i \cdot k(x, x_i)\right) = K(x, y)$$

5.3 Validation and Evaluation

The data set is divided into two different sets. While one is used to build up/train the meta-model, the remaining (test) samples are needed to evaluate the quality of the meta-model according to the Coefficient of Prognosis (COP). Therefore the standard deviations σ of the sample distributions Y_{test} and $Y_{meta\text{-}model}$ as well as the mean value E are needed. The COP ranges between 0 and 1, whereat a COP of 0.5 is equal to 50 % prediction quality [13].

$$COP = \frac{E(Y_{test} \cdot Y_{meta\text{-}model})^2}{a_{Y(test)} \cdot a_{Y(meta\text{-}model)}}$$

For gaining the samples (both for training and testing the meta-model) two different sets of investigations are considered, based on the two different chosen validation methods: Split-Validation and Cross-Validation [12]. The Split-Validation method separates the values in a commonly used ratio of 70 training-samples to 30 test-samples [20]. The 10-fold Cross-Validation separates the data set in a ratio of 90:10. However, the separation is done ten times so that each value belongs once to the test-set. Subsequently, the COPs are calculated for each of the eleven test-samples of each meta-model (ten for Cross-Validation and one for Split-Validation). These analyses were done for the crank mechanism at a crank angle of φ = 310°.

6 Results and discussion

The calculated COP-Values, using the Split- and Cross-Validation for the used meta-models (decision tree, response surface, support vector regression and artificial neural network with varying number of neurons) are diagrammed in figure 3. Due to the different test-values of the Cross-Validation a certain variation of the COP-values can be observed. The prediction qualities of the decision-tree and the response-surfaces are sufficiently high with COPs of 0.9 (equal to 90 %) and higher. The influence of the number of hidden neurons on the neural network's prediction quality can be clearly identified. Too many neurons lead to an over specified system, whereas less neurons lead to an inappropriate fitting of the input data. As a consequence the best results can be achieved using a network with 20 neurons. Nevertheless, concerning the prediction quality the artificial neural network (ANN) with standard (default) parameters has to line up behind the quadratic RSM and the decision tree (Tree).

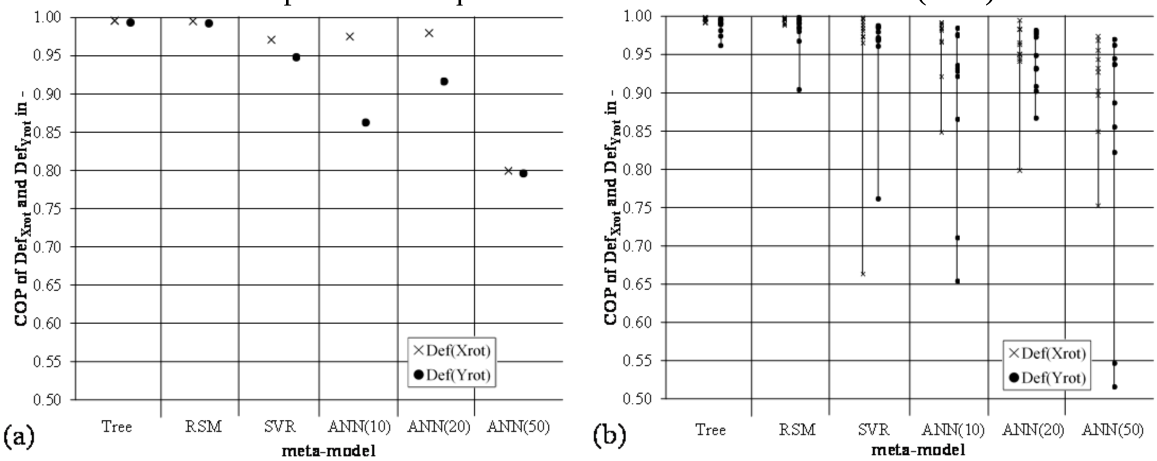

Figure 3 COPs of meta-modeling techniques: Split- (a) and Cross-Validation (b)

Consequently a recommendation can be derived, how the product developer can achieve a high prediction quality of the needed meta-models (from the viewpoint of effort) and therefore an efficient and reliable tolerance analysis of mechanism taking into account the appearing interactions.

The result of the response surface methodology is an exact mathematical relation between the response surface's input and output parameters. Hence, this relationship can be easily integrated into the tolerance analysis functional relation. However, in order to take into account appearing interactions the response surfaces should be at least 2nd order, since these meta-models include interaction terms (so-called mixed terms). If the response surface's prediction quality is not sufficient, there are two possibilities for improvement:

1. Use of additional meta-modeling techniques: As shown in section 6, higher prediction qualities can be achieved by means of additional techniques (e. g. W5-decision trees). However, the definition of the corresponding mathematical relation is either limited to just local relations (W5-decision tree) or mathematically and thus computationally expensive (SVR and ANN). Hence, the response surface methodology should be the product developer's first choice of meta-modeling technique.
2. Variation of the data set: The use of k-fold Cross-Validation instead of a single Split-Validation results in additional prediction quality improvement. However, also the effort increases significantly by the factor k. Hence, the lower the prediction quality (of a meta-modeling technique using a Split-Validation) gets the more effective this recommendation is.

Finally, an appropriate meta-model of the crank shaft's deformation during a motion sequence of the mechanism (consisting of two crank rotations) can be determined using the quadratic RSM (2nd order) of the optimized data set as well as a Split-Validation (figure 4a) and a 10-k Cross-Validation (figure 4b). Figure 4 illustrates the achieved prediction qualities of Def_{Xrot} and Def_{Yrot}, whereas in figure 4b only the cross-validation run with the highest COP is shown.

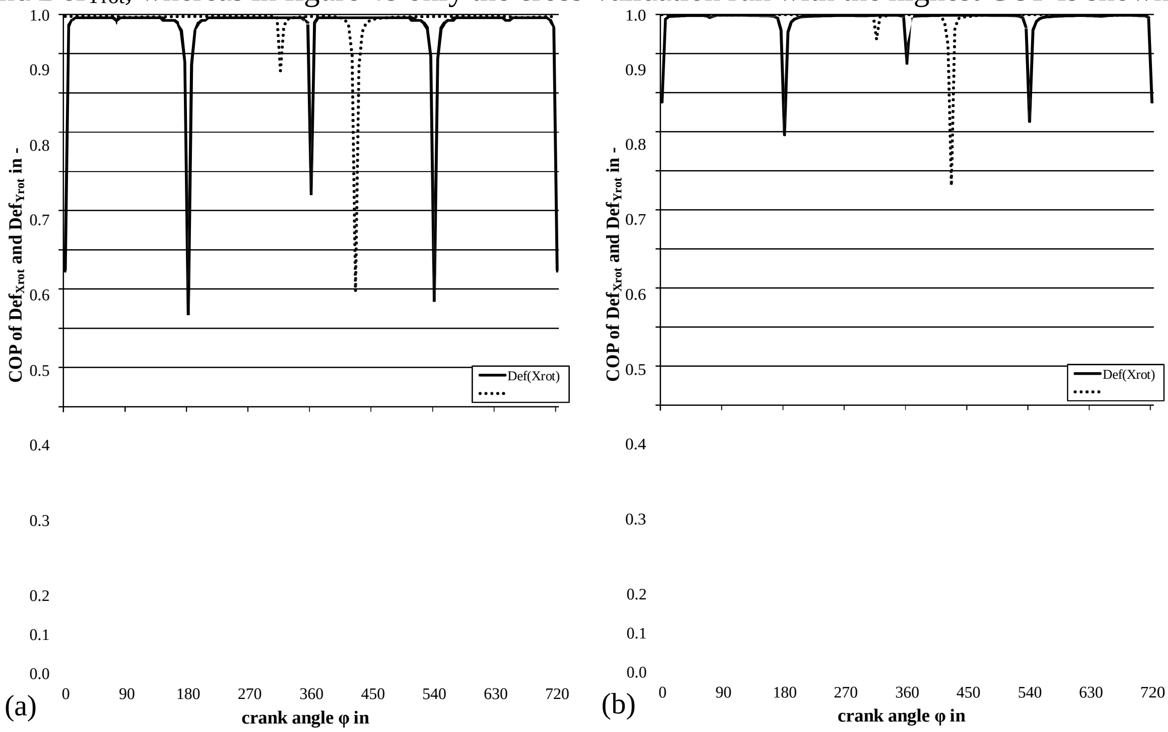

Figure 4 COPs of deformation using Split-Validation (a) and Cross-Validation (b)

7 Summary and outlook

This paper focused on the tolerance analysis of mechanism taking into account the interactions between appearing deviations. Therefore an existing approach – the "integrated tolerance analysis of systems in motion [2]" – was modified by the use of meta-models, representing the operation-depending deviations. Based on an optimized data set, additional meta-modeling techniques were set up and investigated concerning the prediction quality compared to already determined meta-models using response surfaces [3]. The evaluation of each meta-model's prediction quality based on two validation methods: Split-Validation and Cross-Validation. Consequently, a recommendation was derived, how to achieve a meta-model with a sufficient prediction quality and thus a reliable tolerance analysis of mechanism considering appearing interactions. The paper closed with the determination of appropriate meta-models of the crank shaft's deformation with significant higher prediction qualities.

Basically, the extension of the tolerance analysis approach (integration of meta-modeling techniques) enables the product developer to get information about the effects of deviations on a functional key characteristic of a mechanism, as well as the effects of the parameters themselves. But, usually a lot of data is generated during the product development process, which results in an increasing data base. This data can be easily used to constantly improve the meta-models (self-learning effect). However, several meta-modeling techniques (e. g. RSM) can‘t benefit from this effect. Furthermore, the „integrated tolerance analysis of mechanism“ has to be improved towards more complex systems, considering e. g. additional kinds of deviations like thermal effects and wear. Finally, a tolerance optimization can be set up based on the modified functional relationship of a time-depending mechanism in use.

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