

DESIGN METHOD AND TAXONOMY OF OPTIMIZED REGULAR CELLULAR STRUCTURES FOR ADDITIVE MANUFACTURING TECHNOLOGIES

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Abstract

Additive manufacturing technologies enable the fabrication of innovative parts not achievable by other technologies, such as cellular structures, characterized by lightness and good mechanical properties. In this paper a novel modeling and optimization method is proposed to design regular cellular structures. The approach is based on generative modeling of a structure by repeating a unit cell inside a solid model, obtaining a beam model, and on an iterative variation of the size of each section in order to get the desired utilization for each beam. Different structures have been investigated, derived by six cell types in two load conditions. Taxonomy of cell types as a function of relative density and compliance were proposed in order to support the design process for additive manufacturing of cellular structures.

Keywords: Design Methods, Simulation, Computer aided design (CAD), Cellular structure, Additive manufacturing

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1 INTRODUCTION

New opportunities in the diffusion of regular cellular structures come from Additive Manufacturing (AM) technologies that allow the fabrication of parts directly from the 3D virtual model. Therefore AM avoids the limitations that affect traditional manufacturing methods (Gibson et al. 2010, Cerardi et al. 2013) allowing the production of geometrically complex parts without increasing the process complexity. Moreover AM enables the fabrication with high energetic productivity (Franco et al. 2010).

Depending on base material and cells characteristics, cellular materials can achieve high stiffness or strength per unit mass and can provide good energy absorption, good thermal and acoustic insulation properties (Gibson and Ashby 1997, Evans et al. 1999).

Regular cellular structures enable more control on material distribution than foams (Evans et al. 2001). Due to this, structures with controlled stiffness can be designed. In several application fields, like the biomedical one, where the design goals are weight and stiffness of the part, the homogenization method can be applied (Parthasarathy et al. 2011, Fang et al. 2005, Sun et al. 2005, Hutmacher et al. 2004). In this approach a Representative Volume Element (RVE) is selected to represent the cellular structure of the implant, and the mechanical properties, such as the effective Young's and shear modulus, are then calculated. The mechanical properties of the RVE can be changed varying shape and dimensions of the base cell. Once the effective mechanical properties are determined for a range of structures with different porosities, one with the desired characteristics is selected to design the part. For assessing the performance of the part, FEM analysis can be carry out using the RVE mechanical properties as material properties input.

In the field of Computer-Aided Design for Additive Manufacturing (DFAM) for general structural applications (Rosen 2007), a design synthesis method for regular cellular structures is presented in the work of Chu et al. (2010). For a regular cellular structure, formed by interconnected cylindrical elements, under a specified loading condition, values of the section radius of the elements that minimize the volume of the structure for defined stiffness are sought. The sizing problem is solved performing the minimization of a multi-objective function: the weighted sum of goals deviations from target nodal deflection and volume of the structure. A gradient based algorithm that performs a least-squares minimization, Levenburg-Marquardt (LM), is suggested.

The main goal of this work is to propose a Computer-Aided method for generative design and optimization of regular cellular structures, obtained by repeating a unit cell inside a volume, where the elements are cylinders having different radius. The approach is based on iterative variation of the radius in order to obtain the desired utilization for each element. Minor aim is to support the designer in the selection of the better cell type based on the desired functional characteristics of the designed part manufactured by AM technologies. This target has been achieved providing taxonomy of different cell types as a function of the load condition, the structure density and a structure compliance index (inverse of stiffness).

2 DESIGN METHOD

The proposed design method (figure 1) is based on the substitution of a solid model with cellular structures, obtaining a wire model computed by a generative modeling approach (Rutten 2014). A finite element (FE) model is built on the wire model and then analyzed (Presinger 2014). A dedicated iterative optimization procedure was developed in Python (McNeel 2014) in order to obtain an optimized geometric model.

The wire model is assembled by repeating side by side a regular unit cell of specified dimension, in the bounding box of the solid model. Then the lines or part of lines outside the solid model were removed. Each type of unit cell is defined by a number of lines and consequently the wire model is a collection of lines topologically connected at points called nodes.

Each line of the wire model is a beam of FE model. In order to avoid anisotropic effects due to orientation of the cross-sections, in this study all the beams have circular sections. Pipe elements are not used because they do not induce any saving of material-costs in AM technologies due to the difficulties emptying the structures. The radius adopted in the first iteration was the same for each beam and it ensured the desired value of utilization for the most stressed beam. Similar to the inverse ratio of the "factor of safety", the utilization index specifies the level of usage of the material for an element (utilization is equal to zero when the load is null and is equal to 1 when the maximum stress inside an

element is the maximum admissible, i.e. equal to yield stress). Utilization calculation approximates the procedure outlined in Eurocode 3 (CEN, 2007), taking into account axial force, biaxial bending, torsion, shear force and buckling load in compression (Presinger 2013). To complete the FE model, material, load and supports must be defined according to functional requirements of the solid model.

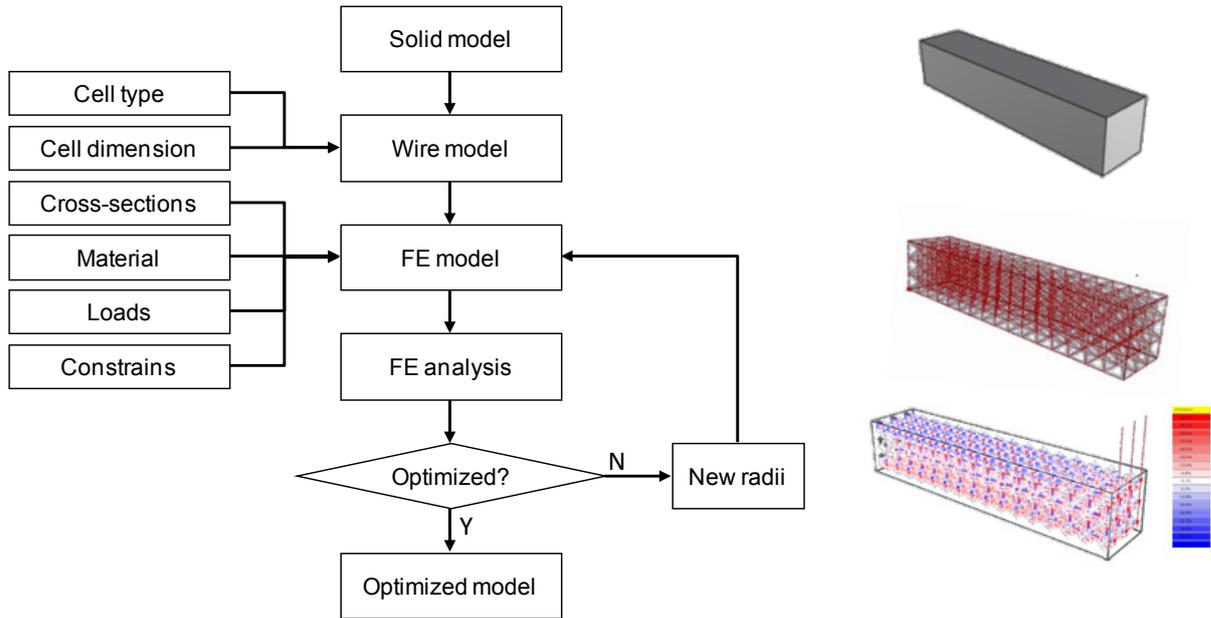


Figure 1. Schematic representation of the design method

The most important result of FE analysis is the utilization of each beam (U_i = utilization of i-th beam), needed in the optimization step. Goal of the optimization is to obtain U_i of all beams close to a target utilization U_t . In order to consider the AM process features, a minimum radius (R_{min}) for each beam must be defined; moreover a max radius (R_{max}) is computed considering the cell dimension.

More in detail, the optimization procedure consists of an iterative modification of the radius R_i of each beam (therefore defining a new FE model) and involves new results of the FE analysis. Each new radius R_{n_i} is defined as:

$$R_{n_i} = R_i \cdot \sqrt{\frac{U_i}{U_t}} \quad (1)$$

$$\text{if } R_{n_i} > R_{max} \text{ then } R_{n_i} = R_{max} \quad (2)$$

$$\text{if } R_{n_i} < R_{min} \text{ then } R_{n_i} = R_{min} \quad (3)$$

The derivation of equation (1) is explained in appendix.

The iterative procedure continues until U_i of each beam satisfies the following equation:

$$U_t - x \cdot U_t < U_i < U_t + x \cdot U_t \quad (0 < U_t, x < 1) \quad (4)$$

where x is the relative deviation of U_t accepted, or until a beam radius R_i

$$R_i > R_{max} \quad (5)$$

If the last condition is satisfied, then the previous iteration is taken as optimal results.

Finally, the optimized geometrical model is computed: a cylinder having the optimized radius and spherical caps is constructed around each line of the wire model. Then, a boolean union is carried out over all cylinders. Spherical caps are adopted in order to reduce stress concentrations and to avoid non-manifold entities at the nodes, where several beams having different radius converge together. A similar approach was proposed by Wang et al. (2005).

3 RESULTS AND CONCLUSIONS

3.1 Test cases

A cantilever beam with dimensions 30x30x80 mm was adopted as solid model to optimize. The cantilever beam was filled with 6 types of cubic cells (figure 2) having dimension of the edge equal to 10 mm: simple cubic (SC) (Luxner et al. 2005), body center cubic (BCC) (Luxner et al. 2005), reinforced body center cubic (RBCC) (Luxner et al. 2005), octet truss (OT) (Deshpande et al. 2001), modified Gibson-Ashby (GA) (Roberts and Garboczi 2002), modified Wallach-Gibson (WG) (Wallach and Gibson 2001). WG cell is asymmetric and the volume was filled iteratively mirroring the shell shape along the propagation direction obtaining a wire model as in figure 3. Each beam of GA has a length equal to one-fourth of cell edge dimension.

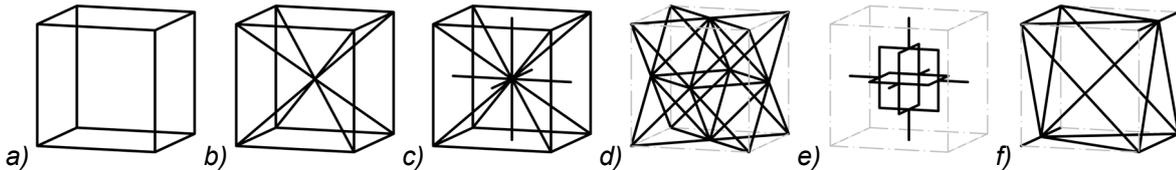


Figure 2. Cell types: a) SC, b) BCC, c) RBCC, d) OT, e) GA, f) WG.

Two separate cantilever beam load conditions were applied to the FE model:

- 50 N vertical force at the free end of the cantilever,
- 50 N axial compressive force at the free end of the cantilever.

The load force was distributed in the nodes of the beams lying in the free end of the cantilever beam. The nodes of the beams lying in the fixed face of the cantilever were adopted as fixed supports.

Since the WG cell type is asymmetric, the cell orientation adopted during the volume fill step influences the results related to the load condition. Figure 3 show the orientation adopted referred to the load orientation. The WG and RBCC free ends were not loaded.

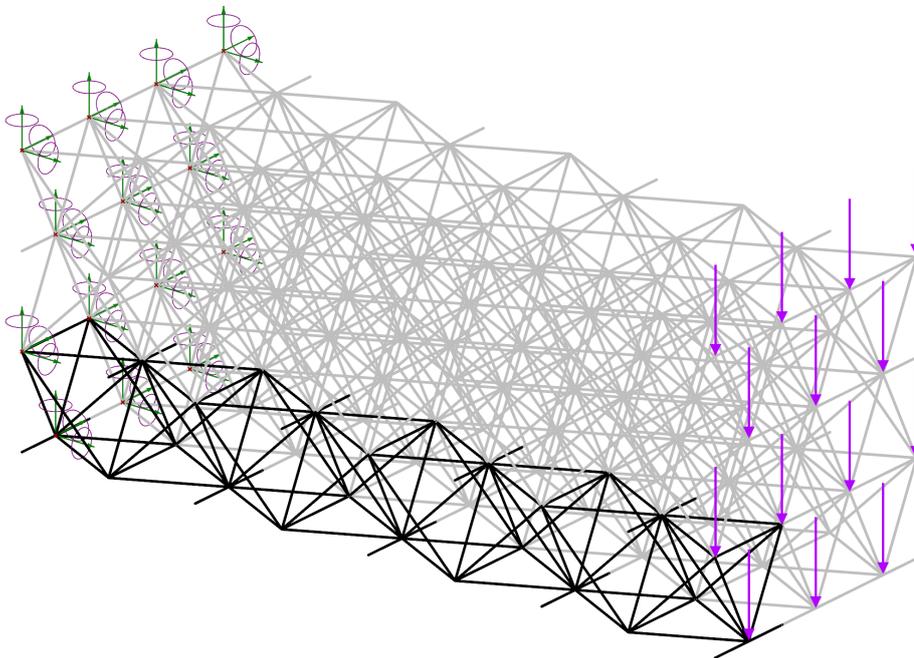


Figure 3. WG studied structure vertical loaded

The material used in the design and optimization process is polyamide 12 (PA 2200 by EOS GmbH) having the following mechanical properties: tensile modulus $E=1700$ MPa, yield strength=48 MPa, shear modulus $G=630$ MPa, density 930kg/m^3 (Amado-Becker et al. 2008).

The convergence conditions adopted in the iterative procedure are: $U_t=0.5$, $x=0.10$ ($0.45 < U_i < 0.55$), $R_{\min} = 0.3$ mm, $R_{\max} = 2$ mm.

To compare the cell types performance, relative density and compliance index were computed in different load conditions.

Relative density ρ was assumed as

$$\rho = \frac{V_o - V_c}{V_c} \quad (6)$$

where V_o is the volume of the optimized structure and V_c is the volume of the cantilever ($V_c = 30 \times 30 \times 80 = 72000 \text{ mm}^3$).

Compliance index D was computed as:

$$D = \frac{d}{F} \quad (7)$$

where d is the maximum displacement and F is the load force.

The optimized structures were compared with cellular structures having a constant radius that ensure the desired value of utilization for the most stressed beam.

3.2 Results

An example of optimized BCC structure showing utilization and the nodal displacement under vertical force is shown in figure 4.

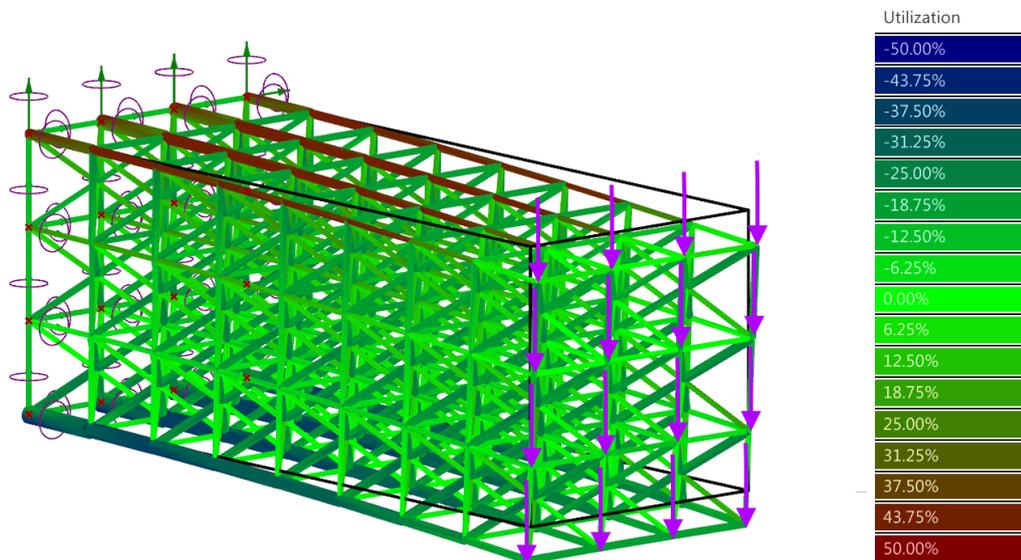


Figure 4. BCC optimized structure under vertical load: utilization and displacement.

All the test cases converge to solution, i.e. all the beam of the structures, except those with minimum radius, have utilization ranging between 45% and 55%.

Number of beams, initial radius and iterations needed to find a solution for vertical and axial load conditions, obtained adopting different cell types, are indicated in table 1. Results are sorted by number of beams.

Table 1. Number of beams, initial radii and iterations, for vertical and axial load condition, adopting different cell types

	SC	WG	BCC	RBCC	OT	GA
Number of beams	344	696	920	1193	1956	2001
Initial radius (vertical load)	1.2	0.725	0.875	0.825	0.725	1.35
Number of iterations (vertical load)	49	19	20	14	22	39
Initial radius (axial load)	0.485	0.504	0.47	0.464	0.342	0.515
Number of iterations (axial load)	1	24	6	12	4	6

Figure 5 shows relative density ρ of the structures obtained by different cell types, in both load conditions. Relative density is computed either before the optimization, when all the beams have the

same radius (CS), and after the optimization (OS). The optimized results are sorted in ascending order to define a ranking among cell types for each load condition.

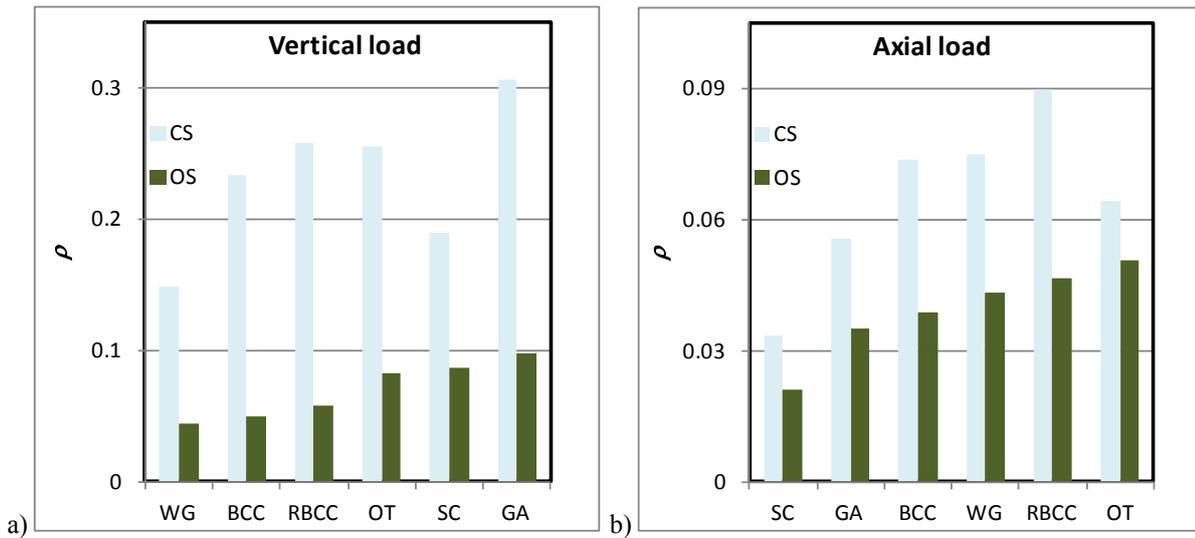


Figure 5. Relative density ρ of the structures obtained by different cell types before the optimization (CS) and after optimization (OS): a) under vertical load, b) under compression.

Figure 6 shows compliance index D of the structures obtained by different cell types, in both load conditions. Compliance index D is computed either before the optimization, when all the beams have the same radius (CS), and after the optimization (OS). The optimized results are sorted in descending order to define a ranking among cell types for each load condition.

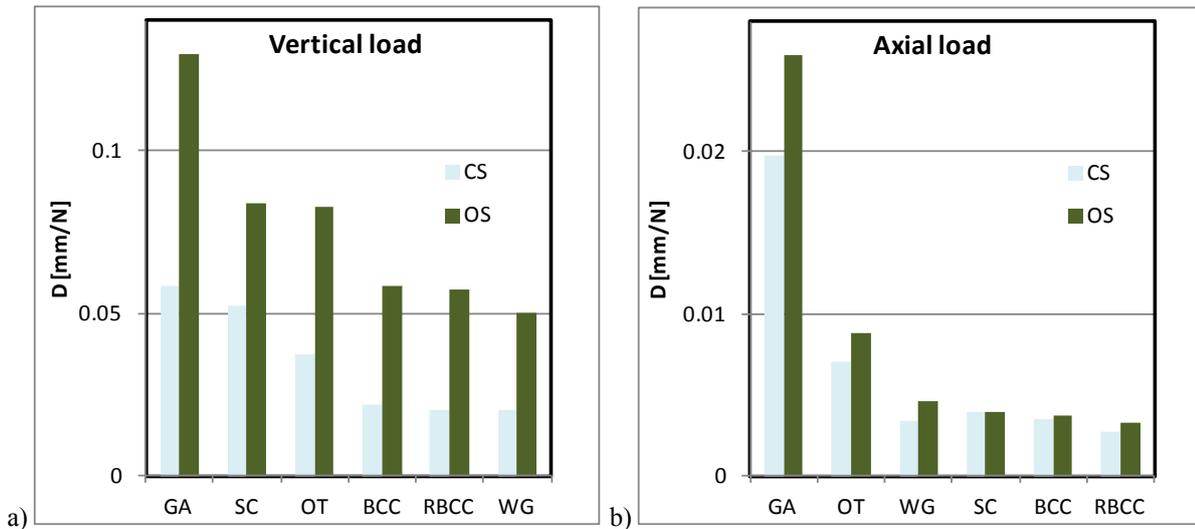


Figure 6. Compliance index D of the structures obtained by different cell types before the optimization (CS) and after optimization (OS): a) under vertical load, b) under compression.

3.3 Discussions and conclusions

Number of beams is closely related to the cell complexity and to the overlap with the adjacent cells. For instance, a SC cell consist of only 12 beams and all are overlapped in adjacent cells (this means that each internal beam lies on 4 adjacent cells): consequently the total number of beams is low. OT has 36 beams per cell, 24 of them are overlapped, while GA has 30 beams per cell, and only 6 common beams (the free ends of each cell are joined together with the free ends of the adjacent cells in a common beam): consequently the GA structure has more beams than the OT one.

No relation appears between number of elements and number of iterations needed to optimize the structures. Only in the case of axial load, adopting a SC cell, a unique iteration is needed, since all the beams subjected to a force are axial loaded (see appendix).

Disregarding the number of iterations, which does not show a clear trend, time needed to optimize the structure and complexity of the geometric model increase with the number of beams. Therefore the order proposed in table 1 could be adopted in the design phase to select a cell type depending on the desired computational speed and lightness of the geometric model.

Figure 5 shows that the optimization procedure always gives a reduction of the relative density. Particularly regarding the vertical load condition and to the BCC and RBCC cell types.

The structure that allows the minimum relative density under vertical load is based on WG cell oriented in the proposed configuration (figure 3), while the minimum density under axial load is obtained with the SC cell. A good balance of lightness in both the load conditions can be obtained adopting BCC or WG cells.

The optimization procedure increases the compliance index (figure 6) especially adopting RBCC and BCC cells under vertical load.

If the functional requirement is related to a compliant material, GA cell structures give the best solution. Different, if the stiffness is the design target, RBCC and BCC cell structures are recommended. WG cell too gives good stiffness in the proposed configuration (figure 3) under vertical load.

An optimization approach based on genetic algorithms has been attempted although the results were not encouraging; similar results has been obtained by Chu et al. (2010) adopting particle swarm optimization techniques.

Several approaches were proposed in literature for structural topology optimization (Rozvany 2009). A number of papers speak about the mechanical properties of different open regular cellular structures as a function of the relative density, studying the problem as beam or as brick structures (e.g. Khaderi et al. 2014, Cerardi et al. 2013, Ramirez et al. 2011, Roberts et al. 2002, Wallach et al. 2001). These could be adopted in the structural optimization, changing, in the best case, the local density of a selected cell type within the solid. At our knowledge, the proposed synthesis method is the first that allows the design of optimized 3D regular cellular structures, ensuring a specific structural strength for each beam. Previous studies (Chu et al. 2010) follow a similar approach, but the optimization function was based on nodal deflection and volume of the structure.

Numerical results give guide lines in the selection of the cell type according to the functional and economical design requirements such as compliance (or stiffness) for an established load configuration, relative density (volume of material), design time and geometrical model complexity.

Future works include the investigation of other load configurations, WG and GA aspects ratio and WG orientation. Moreover, the numerical results need to be confirmed by experimental tests. Finally, it could be interesting to investigate the effects of shell boundary introduction in the optimization method and in the geometric modeling procedure.

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APPENDIX: OPTIMIZATION EQUATION

In this appendix, the derivation of the equation (1) is explained. Considering a simple cylindrical beam under tensile load, stress σ and utilization U are defined as:

$$\sigma = \frac{F}{\pi \cdot R^2} \quad U = \frac{\sigma}{Y} = \frac{F}{\pi \cdot R^2 \cdot Y} \quad (8)$$

where Y is the yield stress of the material, R is the beam radius and F is the axial load.

Consequently, the radius R that ensures an established utilization is given by:

$$R = \sqrt{\frac{F}{\pi \cdot U \cdot Y}} \quad (9)$$

and the radius R_n needed to obtain a target utilization U_t is equal to:

$$R_n = \sqrt{\frac{F}{\pi \cdot U_t \cdot Y}} \quad (10)$$

The ratio between R_n and R is:

$$\frac{R_n}{R} = \frac{\sqrt{\frac{F}{\pi \cdot U_t \cdot Y}}}{\sqrt{\frac{F}{\pi \cdot U \cdot Y}}} = \sqrt{\frac{U}{U_t}} \quad (11)$$

Consequently we can adopt a new radius in the iterations of the proposed method, that allows to achieve the target utilization U_t by the equation:

$$R_n = R \cdot \sqrt{\frac{U}{U_t}} \quad (12)$$

The optimization approach clearly works better when all beams are loaded along their axes. In different load condition, this approach still allows to achieve the optimization although through multiple iterations. Moreover, when a single cross-section is modified, all the elements of the structure change their deformation, stress and therefore utilization; consequently only an iterative procedure can find an optimized result for whole structure.

