



NUMERICAL APPROACH FOR FATIGUE INITIATION SERVICE LIFE OF MECHANICAL ELEMENTS

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1. Introduction

Engineering analyses of material fatigue started by Wöhler (S-N curves) in the middle of the 19th century, followed by Bauschinger (Bauschinger's effect), Kirsch (stress intensity factor) and a bit later by Palmgren and Miner (hypothesis of fatigue damage accumulation). Then, in 1930's, the development of dislocation theory (Orowan, Polanyi, Taylor) proved to be the basis for understanding the process of metal material fatigue. The works of Bilby, Cottrell and Swinden (simulation of dislocation motion in the micro-level of the material structure) and their extended model, elaborated by Navarro and Rios (short and long fatigue crack growth model) proved to be of utmost importance [Navarro and Rios 1988]. It is beyond doubt that the works [Mura and Nakasone 1990] and [Cheng et al. 1994] represent a large step forward in the field of developing physical and mathematical models of fatigue damage initiation. The applicability of analytical methods is limited to idealised engineering problems. It is based upon well known theoretical methods for determining fatigue damage initiation: Coffin-Manson's hypothesis, Morris's analysis, Goodman-Gerber's method, etc. Modern approaches to fatigue initiation analyses are based upon the method which determines the ratio between the specific deformation and the number of loading cycles, often referred to as local stress-strain method [Šraml 2001]. Lately, a connection between the results, obtained by means of the finite element method (FEM) and/or the boundary element method (BEM), and those, obtained by the analyses of material fatigue [MSC/Fatigue 1999], [Šraml 2001] can be traced. The connection between numerical results and those obtained by experimentally determined fatigue parameters represents the basis for determining the changes in the micro-level of the material structure which causes material hardening or softening and the phenomenon of residual stresses (tensile, compressive). Nowadays, the significance of dealing with numerical analysis of different mechanical elements and constructions [Ekberg 2000], [Flašker et al. 2000], [Šraml and Potrč 2000] should be of great importance.

The process of fatigue is a material characteristic and depends upon cyclic plasticity, local deformation, dislocation, formation of micro- and macro-cracks and their propagation. Contact fatigue is of importance in engineering applications where specific contact loads appear (e.g. gears, rolling bearings, wheels/rails). This results in stresses, which can be equal to or lower than the yield stress of a certain material. In respect to operating conditions, there are, generally, two contact fatigue failures: surface fatigue and subsurface fatigue. Material microstructure has a very important influence upon contact fatigue. The characteristics of material fatigue depend upon the presence of dislocations, inclusions and processes going on in crystal boundaries and upon other, not yet fully researched, influences. The experiments carried out so far have shown that dislocation motion on certain slip planes leads to local plastic deformation, which results in surface wear, due to sliding, and in initiation of contact fatigue cracks. The accumulation of dislocations in crystal boundaries, along interstitial or

substitutional atoms and various inclusions, causes accumulation of deformation energy. Under the condition of sufficient energy level, this causes the formation of a new surface in the crystal structure – i.e. micro-cracks. Due to external loading, those micro-cracks propagate, they connect and, gradually, macro-cracks are formed. The propagation of microstructural and physically short cracks in metal materials is, generally speaking, a very complicated process. The main characteristic of the propagation of short fatigue cracks is their discontinuous character as they are, to a large extent, influenced by material microstructure. A certain quantity of accumulated deformation energy in a slip plane of individual crystals is required for the slip plane to propagate into the next crystal grain. When crack propagation reaches a certain number of crystal grains, the level of crack propagation achieves a more continuous character. Concerning to longer cracks, of the order of magnitude of several tens of crystal grains, it is no longer required to consider the influence of material microstructure. In such cases, the theory of linear fracture elastomechanics, which deals with crack propagation in homogeneous materials, is usually applied.

Contact fatigue process can be divided into two main parts:

- (i) initiation of micro-cracks due to local accumulation of dislocations, high stresses in local points, plastic deformation around inhomogeneous inclusions or other imperfections in or under contact surface;
- (ii) crack propagation, which causes permanent damage to a mechanical element, i.e. the exceeding of fracture toughness of the material.

First stage of service life at contact fatigue of mechanical elements, i.e. initiation of contact fatigue cracks is the main concern in present paper. The purpose of present study is to elaborate a new numerical model for prediction of contact fatigue initiation, based on continuum mechanics, which will include adequate loading cycles and characteristic material fatigue parameters. The repeated rolling and/or sliding contact loads cause permanent damages in the material, a fatigue process, due to accumulation of plastic deformation. Finally, the influence of friction on the contact loading cycles and fatigue initiation should be examined.

2. General numerical model of contact fatigue crack initiation

Comprehensive model for contact fatigue life prediction of mechanical elements should consider time history of applied contact loads, regarding both magnitude and position. Typically mechanical elements to attribute contact fatigue process, such as gears, wheels and rails, rolling bearings etc., are providing rolling-sliding contact loads, which are usually stochastic, due to geometric and loading changes at contact.

2.1 Determination of loading cycle and critical plane at repeated contact loading

Dealing with general case of contact loading, the normal as well as tangential forces were presumed for determination of surface and subsurface contact stresses. An elastic and isotropic material model, without material imperfections or damages (Figure 1) was assumed. On the basis of contact stress analysis with modified Hertzian boundary conditions, a loading cycle for each observed material point y_i in/or under the contact surface is determined. In such a way, a realistic description of a cyclic rolling and/or sliding contact load in time domain was achieved [Šraml 2001]. The basic idea for determining loading cycle at general case of contact fatigue, dealing with rolling-sliding boundary conditions, is presented in Figure 1. The procedure of numerical calculation can be described as follows:

- (i) For each material point y_i at the contact region (Figure 1) the loading cycle was determined in response to moving contact load at time domain t . Using the equivalent contact model of two cylinders, the generalised Hertzian contact theory is applied. At first, the proper boundary conditions have to be prescribed and equivalent parameters have to be calculated (equivalent radius of contact bodies, elastic modulus, Poisson ratio etc.).
- (ii) Hertzian theory is used to evaluate boundary conditions for contact stress analysis, which were performed in the framework of the finite element method. In this manner the influence of stress field at contact region for each observed material point y_i was defined. This procedure lead us to loading cycles at rolling-sliding boundary conditions, which will be further on used for contact fatigue analysis.

(iii) Finite element model, used for calculation of contact stresses and determination of loading cycles, is shown on the Figure 2. Following boundary conditions, geometric and material data were used:

- Maximum Hertzian pressure: $p_0 = 1550 \text{ MPa}$;
- Elastic material module: $E_1 = E_2 = 2,07 \text{ GPa}$;
- Poisson ratio: $\nu_1 = \nu_2 = 0,3$;
- Contact length: $2a = 0,5476 \text{ mm}$;
- Coefficient of friction: $\mu = 0,0; 0,1; 0,2; 0,3; 0,4; 0,5$.

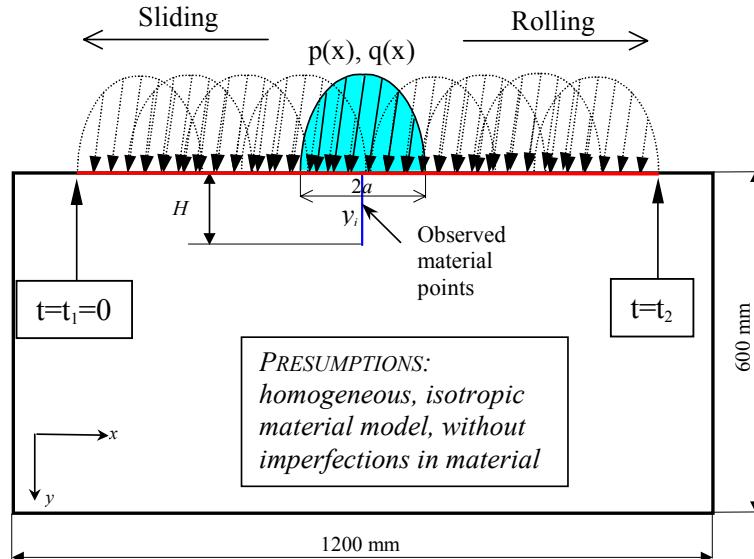


Figure 1. General idea for determination of loading cycle at contact fatigue of mechanical elements

Material model is prescribed with cyclic material curve, which presents relation between stress range $\Delta\sigma$ and plastic deformation range $\Delta\varepsilon_p$ and can be expressed as [Zahavi 1996], [Suresh 1998]

$$(\Delta\sigma) = K' (\Delta\varepsilon_p)^{n'}, \quad (1)$$

where K' is strength coefficient and n' is hardening exponent.

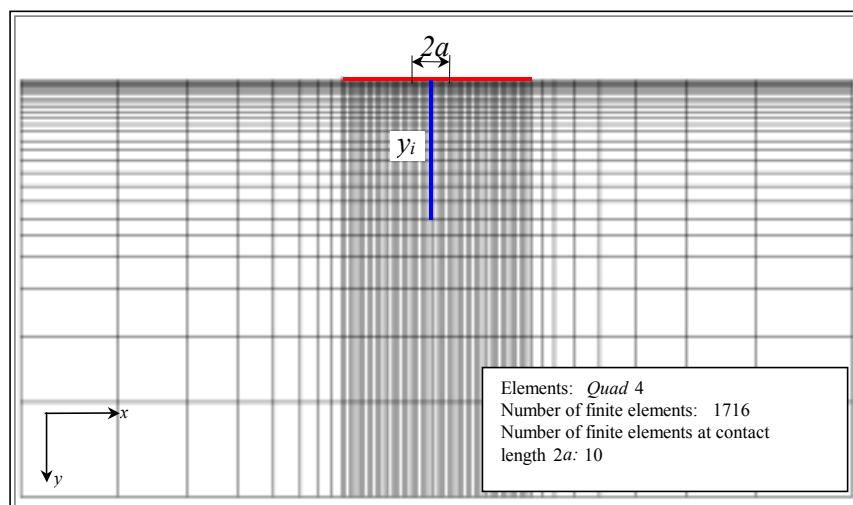


Figure 2. Finite element model for determination of contact loading cycles

The influence of friction upon the stress cycles is dealt with in detail. Considering the fact that contact fatigue is, to a large extent, influenced by contact surface roughness and friction (regardless of its origin), the influence of friction on the course of a loading cycle and also its material fatigue process have been analysed. Thus, the fundamental results in the suggested model are as follows: the course of standardized loading cycles, in respect to various stress components (σ_x , σ_y , τ_{xy}), comparative stresses Tresca (Figure 3), the largest principal stresses (σ_1) and hydrostatic stresses (σ_h).

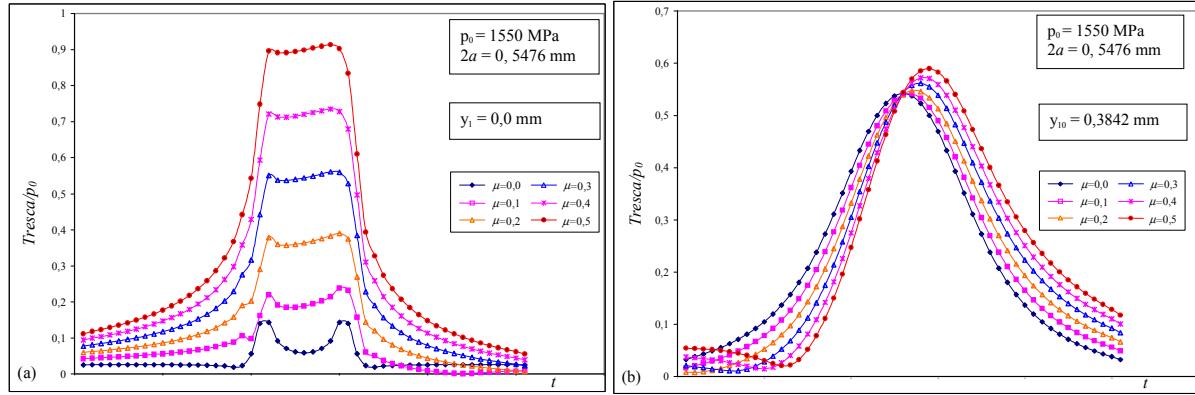


Figure 3. Loading cycles based on Tresca comparative stresses:
(a) on the contact surface y_1 ; (b) subsurface y_{10}

The forms of loading cycles and their characteristic values are of significant importance for further contact fatigue analysis.

2.2 Determination of loading cycle at repeated contact loading*

After the determination of loading cycles, the fatigue initiation analysis is dealt with in detail. Methods of fatigue analyses are most frequently based on Coffin-Manson relation between deformations, stresses and the number of loading cycles (eq. 2), [Zahavi 1996], [Suresh 1998]

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_a}{E} + \frac{\Delta\epsilon_p}{2} = \frac{\sigma_f'}{E} (2N_f')^b + \epsilon_f' (2N_f')^c. \quad (2)$$

Modified forms of the fundamental Coffin-Mason model include the influences of mean values of stresses (σ_m) and of other material parameters – fatigue strength coefficient (σ_f') and fatigue ductility coefficient (ϵ_f'), exponent of strength (b) and fatigue ductility exponent (c). Previously determined loading cycles (Figure 3) have to be connected with corresponding material fatigue models. Thus, the following three methods were taken into consideration: ϵ - N , Smith-Watson-Topper (SWT) and Morrow method which are, generally speaking, based upon the relation between deformation and the number of loading cycles, required for fatigue damage initiation [Suresh 1998].

The most frequently used method for calculating number of cycles, needed for fatigue crack to occur, is based on strain-life method (ϵ - N). Material curve (Figure 4) can be fully characterized by knowing four, previously described material parameters σ_f' , b , ϵ_f' , c as shown in the equation (2). This curve consists of an elastic component and a plastic component, which can also be plotted separately. The transition point T (Figure 4) defines the boundary between high cycles fatigue (HCF) versus low cycles fatigue (LCF). All the fatigue material parameters were taken from [MSC/Corporation 1999].

2.3 Results for number of loading cycles of contact fatigue crack initiation

As a general example, the present model for analysis of fundamental contact problem of a cylinder and flat surface was used, because of the comparativeness of the results with those of the existing models [Cheng et al. 1994], [Mura and Nakasone 1990], presented and used in work [Glodež et al. 1999].

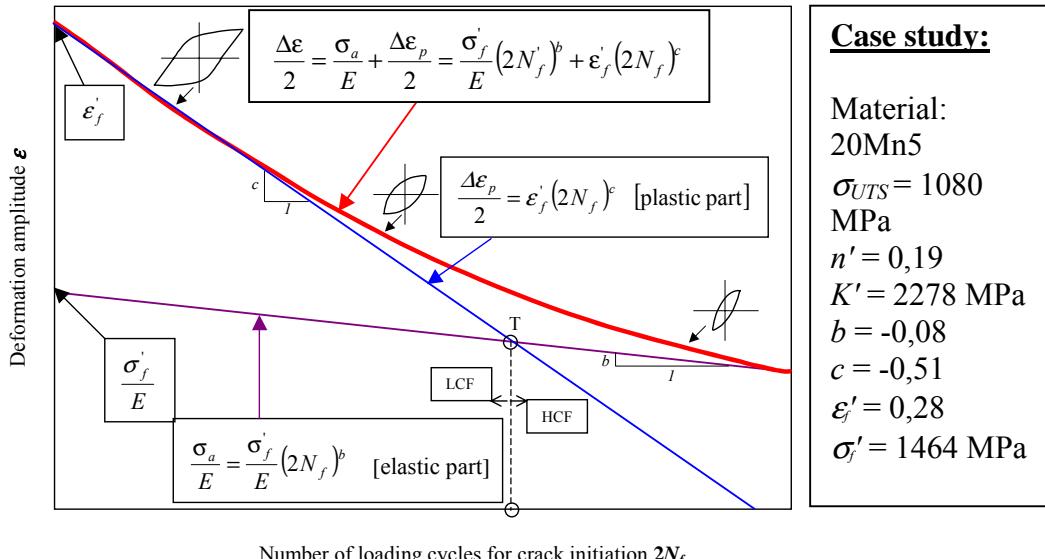


Figure 4. Strain-life (ϵ -N) material curve and material properties [MSC/Corporation 1999]

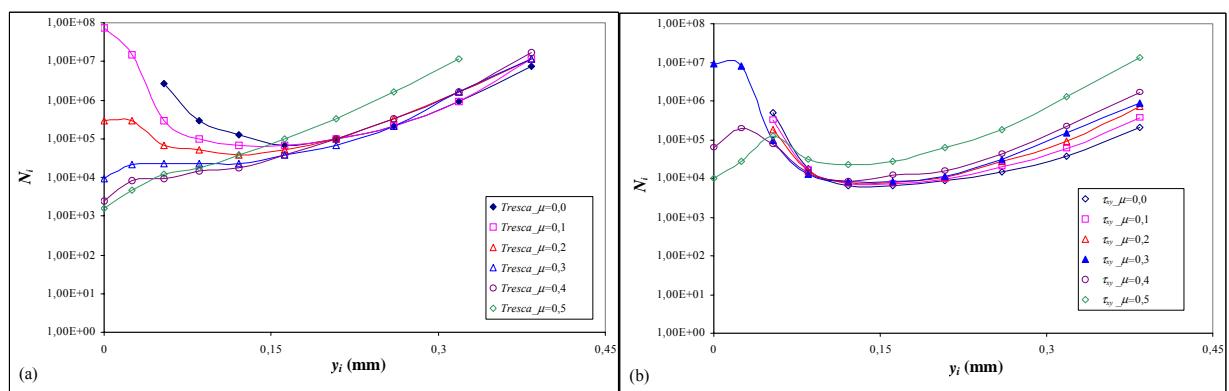


Figure 5. The number of loading cycles for fatigue crack initiation:
(a) comparative Tresca based loading cycle; (b) shear stresses τ_{xy} based loading cycle

Following examples of contact loading cycles concern equivalent Tresca based loading cycles and shear stress based loading cycles. When dealing with effective Tresca based loading cycle, the number and place (y_i) of loading cycle for crack initiation (Figure 5 (a)) depends on coefficient of friction μ . In the case of $\mu=0,0$ number of loading cycles required for initial fatigue damages to appear is $N_i=69813$ and first occur at point $y=0,16193$ mm under contact surface (Figure 5 (a)). Highest coefficient of friction means that the number of cycles N_i reduce and the place y_i is more in the direction towards the contact surface. However, when dealing with shear stresses τ_{xy} based loading cycle (Figure 5 (b)), the number of loading cycles required for initial fatigue damages to appear in the case of $\mu=0,0$, is $N_i=6289$ and first occur at point $y=0,12126$ mm under contact surface, respectively. Extreme values of coefficient of friction ($\mu=0,4$ and $\mu=0,5$) cause the fatigue damage occur relatively early in the fatigue process and its place is as a rule on the contact surface.

It was established, that regardless of selected stress component, the number of loading cycles, required for initial fatigue damages, is in the range of $N_i = 10^4 \div 10^7$ and where (on the contact surface or subsurface initiated contact fatigue) the contact fatigue damage first occur mostly depends on the coefficient of friction (limit value is about 0,3), material parameters and contact geometry.

3. Conclusions

The paper presents a numerical model of fatigue damage initiation due to contact loading of mechanical elements. This new model enables the determination of the number of loading cycles N_i , required for fatigue damage initiation, by means of certain loading cycles and adequate material fatigue parameters. The method of dealing with numerical modelling and the possibility of predicting fatigue damage initiation in mechanical elements as a consequence of cyclic contact loading represent major contribution to the field of contact fatigue initiation. Rolling-sliding boundary conditions on the contact surface are taken into consideration mainly throughout the influence of friction coefficient. The obtained effects of friction influence on contact fatigue initiation are also in good agreement with the fact, that generally, the limit value of friction coefficient for fatigue crack initiated in the surface or under the surface, is 0,3. This mean, that values of friction coefficient higher than 0,3 results in contact fatigue damage initiation, as a rule, on the contact surface. The fact is that presented numerical model enables a better understanding of the process of fatigue crack initiation in contact area, due to moving rolling-sliding contact loading, which is the essential advantage comparison with existed models [Ekberg 2000]. However, the present model can be further improved with additional theoretical and numerical research, although additional experimental results will be required to verify the applicability of the presented and any improved models.

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