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A THEORY OF THE CONSTITUENT ELEMENTS OF FUNCTIONS

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ABSTRACT

The aim of the paper is to present a new proposal for functional analysis. The approach here discussed is the first step towards a novel methodology called FAB (functional analysis breakdown) where the fundamental constituents of functions are investigated. Actually the paper guides the reader towards the discovery of the ultimate indivisible functional elements. The novel approach focuses on how functions act on flow or in other words on the physics, chemistry, mechanics of each function. Such strategy inverts the standard approach, descriptive and taxonomic, usually followed in Functional Analysis. The evidence of the existence of more elementary elements than functions is provided with a series of examples. The paper ends with a research agenda in several fields of application.

Keywords: Functional Analysis, Functional Analysis Breakdown, Augmented Functional Basis

FUNCTIONAL DESCRIPTION

Functional analysis has been historically driven by two different, although intertwined, motivations. On one hand, functional analysis [1][6] is the abstract conceptual basis for a large series of techniques adopted to represent artefacts at various levels of generality, such as QFD [3], value engineering and value analysis [1], Design for [2], TRIZ [4], FMECA [5] and the like. Building a functional base means offering a manageable language to be used in practical applications [9]**Error! Reference source not found.**. In this direction functional analysis has mainly a pragmatic orientation and its performance can be evaluated in terms of generality, completeness, comprehensibility, easiness of use, in solving design problems.

On the other hand, there has always been a deeper level of investigation, one in which functional analysis can be considered itself a tool to produce new and valid knowledge. In this direction, the main problem is ontological: what do we do when we describe an entity in functional terms? Is a functional description equivalent to a physical description? If yes, why do we need another level of description, one whose language and grammar are far less understood? If not, where is the difference? At the heart of this approach there is the difficult issue of the differences, and the relations, between a physical description of reality and a functional description. It is not surprising that these questions have attracted the attention not only of scholars of engineering design, but also of philosophers, physicists, biologists, or computer science theorists.

The purpose of this paper is to develop an approach to functional analysis that explicitly makes the distinction between functional description and physical description, but at the same time makes it possible to define the former in terms of the latter and also to identify the elementary constituents of functions. We will argue that this approach, whose inspiration is clearly theoretic, somewhat paradoxically builds a better foundation for several practical applications.

THEORETICAL BACKGROUND

State of the art in functional analysis in engineering design

In engineering design Functional Analysis (hereinafter FA) played a key role in the formalisation of the design process. It evolved from the original proposals by Miles[1] oriented to cost reduction and the first systematic rationalization by Pahl and Beitz [6] to the interesting efforts of Collins [7], those performed by the NIST [8] and the synthesis proposed by Hirtz et al [9], currently considered a milestone in FA.

The consolidated approach is based on a precise identification of each electromechanical function with a pair of words, a verb plus an object, the former indicating an action and the latter the three possible flows of material, signal, and energy [6]. In this way, even the most complex product can be divided into its functions, i.e. into a series of simple single functions (belonging to ordered lists of abstract functions called Bases) connected to each other via the flows (the inputs and outputs of each function).

The need for a common language in functional descriptions brought many authors to the development of standardised functional lexica, taxonomies, databases, etc. So far many researchers have developed systems based on small functional bases [7][8], following the principle of developing a minimal set of verbs to describe every mechanical function [9]. Such choice is reasonable in the context of a semi-manual use of the basis. The experts would have to decide whether a particular expression should be categorized under a given functional verb in the basis and then choose the right flows.

Such approaches showed many limits of standard FA. These limits have been deeply analysed in [10] and partially solved in [11] through the development of a fully formal functional language.

Other works of great interest for the modelling of the functional space are those of Nagel at al. [17] and Sridharan et al. [18].

Through a suitable syntax and morphology Nagel at al. [17] aim standardization and usage clarification of signal flows, in order to get a more consistent modelling of the sensory elements vital to electromechanical design. This is a first step in a larger effort to move the Functional Basis, currently a structured set of terms describing product functionality, towards a formal language - called by the authors Functional Basis Modelling Language (FBML) [17]. Sridharan and Campbell [18] pointed out that, although functional models are usually constructed from a well-understood black-box description of an artefact, there is no clear approach or formal set of rules that guide the creation of function structures. They carry over the common basis of functions developed by Hirtz et al [9], and define 69 grammar graph rules that capture the feasible set of interactions between these functions. As an analogy to written language, the common basis provided in [9] represents the lexicon of valid terms, while their approach provides the grammar for combining these terms. All these works witness the need for a deeper and richer representation of functions, including both language and grammar and show that the space of functions is indeed much more complex than it has been hypothesized..

The ship of Theseus, or the nature of functional descriptions

It is useful to introduce the problem by referring to an old but still controversial ontological problem, i.e. identity and diversity.

In *Fedone* (58A), Plato talked about the ship used by Theseus, to save himself and seven couples of children. In order to thank Apollo for rescuing the children, Athenians promised to send the very same ship of Theseus each year to Delo, and to refrain from death penalties during the whole period of the journey. To realize this promise, they had to do extensive maintenance of the ship each year and substitute damaged parts. So the problem was- is the ship the very same of Theseus, or rather, piece after piece, the identity changed? What constitutes the identity of the ship, the physical constitution of any single piece of wood, or some other property? This problem has been repeatedly discussed by philosophers, such as Leibniz, Hobbes, Locke and Hume until recent times, in which the problem of identity has been made central to modern ontology (Chisholm, 1976; Hirsch, 1982). From a more engineering perspective such problem has been widely studied by Gero in his numerous papers (e.g. from [13] to [14]), by Bonaccorsi [15] Vermaas[19] and Dorst [20].

The problem is extremely relevant to our discussion, because it sheds light on the dual nature of functions. On one hand, the ship must be made of wood or other material. Without the physical embodiment of the ship, there is no entity called ship. At the same time, describing the ship only in terms of the exact physical constitution is not adequate either, because there is no way to avoid the paradoxical implication that by changing the pieces the ship is no longer the same, which is contrary to evidence. There must be another level of description, one which is compatible with the physical description but is not completely determined by it (Polanyi, 1962). However, in practice functional descriptions are usually done in natural language, or in a loose and purely qualitative way, so that their actual role in producing new knowledge is questionable. If functional descriptions depart from the use of language used in science for describing the state of the world, there is a serious danger of irrelevance. Functional analysis can therefore serve pragmatic goals, as it happens in engineering design, but has no real epistemic value added.

The limits of current functional descriptions

This limitation is not alone, however. Other conceptual problems still plague the current state of the art of functional analysis in various fields, not only in engineering. Take for example biology, a field in which the notion of function is vital, but the dominant view has assumed the existence of a one-to-one mapping between genes, proteins, and functions (Danchin, 1998; Seringhaus and Gerstein, 2008).

Here the identification of the chemical bases of DNA spurred the attempt to define the material constitution of life in purely physical terms, following the early suggestions of Schrodinger (1944) and Feynman (1963). According to the latter, since all things are made by atoms, all what living things do can be understood in terms of movements and oscillations of atoms. But this approach has proved elusive. Information contained in the physical description of constituents of life is not sufficient to explain the properties and behaviours of living things. Thus, although it is true with no exception that "all things are made by atoms", the consequence that all what living things do can be *understood* in terms of atoms is not logically necessary. Indeed, after mapping the human genome, it has been discovered that the relation between genetic materials and biological functions is far more complex than previously assumed. Without any attempt to enter into the complex issue of the theoretical status of functions in biological thought (see among many others, Cummins, 1975; Millikan, 1984; 1989; Sober, 1994), we will use a long citation by Richard Lewontin to elucidate some of the issues at stake. According to Lewontin, the functional description has so far suffered from several limitations.

Our contribution, although originated in engineering not in biology, exactly addresses these issues.

> "One fundamental difficulty in finding the "natural" sutures between parts of an organism is that there are functions at different levels of aggregation. The circulation of the blood serves the vital function of cell respiration by bringing oxygen and removing waste products, so the heart seems a natural anatomic unit.
>
> But the contraction of muscles serves the function of making the heart beat, so the structure of muscle cells and their patterns of innervation is an appropriate level of study. But the shortening of individual muscle fibers serves the function of muscle contraction, and this depends on the chemistry of the proteins actin and myosin. There is a hierarchical cascade of functions that serve other functions above them, and no one of these levels is uniquely correct for the analysis of either the operation or the evolutionary history of the organism. The other problem of function is that in addition to the vertical hierarchy of functions there is a horizontal multiplicity of functional pathways that define parts according to different topologies. Bones serve the function of providing rigidity to the body and attachments for muscles. But they also are the sites for the storage of calcium, and the bone marrow is the tissue within which new red blood cells are produced. Depending on the causal pathway of interest, bones are either macroscopic structural elements or collections of cells that secrete calcium or embryonic tissue of the circulatory system". (Lewontin, 2000, pp. 78-79).

Thus there are two related limitations: the level of analysis is arbitrary, since functions are always part of a larger vertical hierarchy of functions, represented in a tree-like form, and there are overlappings between vertical hierarchies, so that the same structural element can be part of different functional hierarchies.

Summing up, it seems that functional descriptions in several fields of science suffer from the following limitations:

i) *natural language*- functions are described as loosely defined verbal forms, which prevents analytical treatment and rigorous development;
ii) *hierarchical vertical structure*- functions are described in a tree-like form, but there is no logical necessity in identifying the appropriate functional level; above all there is no end in going up and down the vertical hierarchy, making the exercise arbitrary;
iii) *separation of vertical hierarchies*- each function is represented as part of a separate vertical hierarchy, contrary to reality.

We have developed a new functional approach, initially described in [10][11], that is able to address simultaneously these three limitations. The approach is based on a number of new ideas that challenge existing approaches in engineering design and could potentially be expanded to other domains as well. These ideas are the following:

i) use systematically physical language to describe functions;
ii) identify appropriate levels of functional description by relating them to the relevant layer in the physical constitution of the world;
iii) avoid tree-like functional representations and use instead graph-theoretic representations in multi-dimensional vector spaces.

Let us articulate them with respect to the limitations identified above.

FUNCTIONAL DESCRIPTIONS USING PHYSICAL LANGUAGE

The first idea is that functions should always be expressed in such a way to make explicit the physical effect implied. We should always ask "which state of an observable variable is affected by this function" and "which is the change in the observable variable as an effect of the function"?

It is clear that most functions in everyday language are not formulated this way. But this remark applies also to part of the literature, in which functions are expressed as "verb + object", but often the physical content of the verbal or object element is hidden or not at all clear.

Following the early suggestion of Pahl and Beitz [6] we therefore stress the physical transformation (or lack thereof) associated to any function and suggest the following definition.

Definition 1 (Function)

A function is any single action or series of concurring actions that interact with one or more physical characteristics (in the broadest meaning of everything that can be sensed objectively) of the system which is the object of the action (namely the flow).

Some notes about the above definition:

1. The concept of interaction means either changing the state of the system or preventing it from spontaneous or unwanted change.
2. The system, object of the action, includes all the flows which are modified by the action. Both the agent and the object of the action modify their statuses (some of their parameters/characteristics) because of their mutual interaction.
3. The definition is not based on (too) generic flows but it recovers the main physical information about the particular aspects of the flows actually modified by the action.
4. The concept of interaction is strictly linked with the time evolution. In turn, time evolution is derived from physics, where the state of a system is uniquely characterized by a certain set of parameters changing with time due to physical interactions.

This definition focuses on those (preferably) measurable aspects of a flow (material, energy, and signal) that evolve or that are prevented to evolve in time because of the action. What do we mean by physical constitution of the world? Irrespective of epistemological or ontological assumptions, we observe that human-made artefacts, at least before the emergence of nanotechnology, are confined in a relatively small region of size with respect to the known universe (Bonner, 2006). Smallest artefacts are in the order of micrometers, largest artefacts, such as oil pipelines, are in the order of a few thousand kilometres. Within this range, contemporary science offers a relatively complete description of physical and chemical effects. While the physical description is logically and practically *not* equivalent to functional description, still if offers a language that can be used to formalize functions, once they have been identified. It is honest to admit that, at the state of the art of our approach, we claim validity only in the domain of mechanical and control functions, while nothing can (yet) be said on functions that require a physical description in terms of quantum physics, or biological functions, as for example at the nanoscale. Still, the advantage of using the physical description to formalize functions, even in a limited domain, is enormous.

The emphasis on physical language may raise the question of where is the difference between our definition and any proposition in physics or chemistry that describes a phenomenon, or a law of nature. Is our definition a way to make functional descriptions just a specialisation of scientific descriptions? Are we violating Polanyi's principle of independence, or claiming there are no two similar ships of Theseus?

The answer is no. It is clear that the crucial element of the definition is *action*. Causal propositions aimed at describing or explaining phenomena do not give account of actions. To have an action, we need agency, i.e. an intentional action by an artificial agent, or an action produced by the internal functioning of a living or artificial system. Artificial agency and living agency have one crucial element in common: they survive, i.e. they self-repair and self-reproduce: "So what does the appeal to functions actually explain? It explains the existence and properties of those parts of a self-reproducing system that contribute to the self-reproduction of that system. What functions explain is systems whose identity conditions consist in the constant replacement, repair, or reproduction of their component parts" (McLaughlin, 2001, p. 209). The introduction of the action in the definition produces two relevant implications, which we formalize as corollaries.

Corollary 1.1 (Physical language for functional description)
Any function can be fully described with the same language used to describe the physical constitution of the world.

Of course the above corollary does not state that the description used for functions must be identical to that used for physical laws (such as equations and so on). Actually the emphasis given on the concept of action in the definition of function and the complex nature of products and processes that Functional Analysis deal with impose that in most cases the two descriptions will be formulated in substantially different ways. The sense of the corollary is that a parallel between the two alternative descriptions (equations vs functions, to name one) must always be possible.

Corollary 1.2 (Physical language for non functional description)
The same physical language may be used for functional and not functional descriptions.

In a linear feeder the motion of a component along a slope performs the desired function, i.e. to feed. The reference equations (a suitable subset of all equations that could fully describe the phenomenon) are those of kinematics along a tilted plane, those of static mechanics and Coulomb's friction.
The same equations govern the motion of a stone that slips along a hill. But in this case we will not say there is a function. This is because there is no agency, hence no action. From Corollary 1 we also derive a Sub-Corollary that may be useful as a practical rule for developing an adequate language.

Sub- Corollary 1.1.1 (Acceptance criterion for functional description)
Any description, which is done in such a way to make it impossible to identify the relevant physical transformation, is not functional and therefore should be rejected.

Corollary 1.2 makes it explicit that the same physical language can be used in two substantially different ways, one for describing physical structures or laws, the other to describe functions. Polanyi's principle is fully preserved. There will be different syntax and grammar rules, operating however on the same language. The use of physical language is also a condition to advance functional analysis from the current status of pragmatic discipline to a status of scientific discipline, rooted in scientific understanding of the world, but at the same time preserving the fundamental role of action in the world itself. The former (understanding) defines the boundaries of possible worlds, the latter (action) defines the exploration of particular directions in the space of possibilities. A further element can be introduced in functional analysis, i.e. equations. Once we have adopted the physical language to describe functions, there is no logical obstacle to use these highly peculiar representations of relations between physical variables. We limit our definition to the electro-mechanical field, for which the underlying knowledge of physical effects is large and robust. We have developed equation representations for almost all functions in the Augmented Functional Base presented in [11]. In general, equations will not represent functions as such, because they lack the crucial element of action. However, they can legitimately be part of a functional description when coupled with actions. Of course, it will not always be possible to use equations: Definition 2 only states that if a description based on equations is available, it is useful to include equations in the functional description as well.

Definition 2 (Equations)

In the electro-mechanical field, each function can be described on the basis of an equation or of a system of equations. The equations represent the physics, the mechanics, and the overall logics which determine the functional behaviour of an electromechanical apparatus.

In definition 2 we introduce the concept of functional behaviour. This concept covers both expected behaviours (those designed to perform a function) and behaviours of the structure (those derived by the structure of the system). As demonstrated by Gero [13] they can differ a lot. In our opinion the difference between the two can be interpreted in terms of the different subsets of equations that are used to determine the expected behaviour (design) and those that determine it in use.
In physics, equations are used to describe phenomena but the user knows they represents only a part of the real phenomenon. In order to describe the throw of a body we can adopt the throw equations or compute the effect of the air, that of the variation of g with height, the Earth's motion (Coriolis' force) and so on. It ultimately depends on the aim of the analysis. If the aim is to predict the position of a sinker thrown by a fisher along the river, the throw equations are good enough, if the study concerns badminton we need to consider air resistance, to launch a satellite we need a few more equations.
The functional description will therefore include one or more equations plus descriptors of action.

Corollary 2.1 (Origin of the finiteness and diversity of functions)
Since in physics the number of laws is finite (i.e. the number of variables, parameters and operands is finite) the functions that can be described by equations are finite as well. The same parameters with different operands (and the same operands with different parameters) describe different phenomena.

To give a clear example, take cutting processes. Each function describing a cutting process can be represented as a system of several equations. The first one will describe the quantity of material removed per time unit. This equation is basically the same for different processes i.e. milling, drilling, laser cutting, electrochemical machining and so on. Let us label it (eq. i). The subsequent equations will describe and characterise each different process: for example the milling, drilling, turning will follow the same equation that describe the forces acting on the tool, due to the geometry of the tool (cutting angles) and to the hardness of the material (in case of cast iron) or to the tensile strength (in case of steel). That will be labelled (eq. ii). Conversely laser cutting will be described by the equations of heat transfer and evaporation (eq. iii), whose variables are not correlated with those in (eq. ii). Laser cut and milling have in common only equation (i) and not the other one. By characterizing functions by means of equations we can identify similarities across totally different functional problems, hence technological domains and applications. For example, “to blow a flow of air into a tube” and “to move a piece of wood along a straight line” can be properly described by the Bernoulli’s equation (eq. a) and by the motion equation (eq. b), respectively. These equations share a common variable: the speed (v) of the object of the action, but differ for many other parameters. The information on the shared variable can be used to identify solutions in one domain that may be relevant in another domain.

LINKING THE FUNCTIONAL HIERARCHY TO THE PHYSICAL CONSTITUTION OF THE WORLD

The explicit use of physical language is a powerful approach to address the second limitation of current functional analysis, the arbitrariness of functional decomposition. The problem raised by Lewontin in the quotation above, in fact, is a very general one in engineering design. It is well known in the literature on complex problem solving in design, based on cognitive sciences, that multiple decompositions of complex functions are always possible, but that the principles of decomposition tend to be idiosyncratic (Suh, 1990; Baldwin and Clark, 2000). In other words, experienced designers “know” which decompositions are better and adopt decomposition strategies, but this knowledge is not based on formal modelling and therefore cannot be codified and replicated. Consequently, design practice tends to follow successful decompositions previously adopted by others, without any control on their efficiency, creating a condition that the evolutionary literature in economics label path dependency and lock-in (Marengo and Dosi, 2005).

Our suggestion is to link decompositions to the articulation of the material world in layers, a property well known in physics and chemistry, by using an explicit effort to represent functions and sub-functions down to the *most elementary* physical effect available. By explicitly linking functional descriptions to physical variables, it will be possible to locate their place in a multi-layered constitution. Since functional descriptions, however, not only include physical variables but also actions, it will in general be possible to obtain several legitimate hierarchical functional descriptions. This will be true either bottom up (a function as an element of a larger hierarchy of functions) and top down (decomposition of a complex functions in more elementary functions, or sub-functions). However, this multiplicity of decompositions, contrary to the arbitrariness existing in the current state of the art, will permit the identification of transformations from one to another. This also will support the adoption of clear decomposition strategies, such as modularity, or limitation of constraint propagation. In our Definition 3 we will start bottom up and derive the top down movement as a corollary. The opposite would clearly be possible.

Definition 3 (Decomposition)

Any function can be described as an element of one or more hierarchies of functions. At each level of any hierarchy the functional description can be related to a set of physical variables located at a specific level of the constitution of the world.

In the electromechanical domain the levels of physical variables are mainly related to the size of phenomena, although many of the physical variables can remain the same across levels (e.g. speed, metric scale, etc..). In the informational domain the physical foundation is in the electric binary behaviour of electronic circuits. Elementary instructions are built upon binary logic and have a physical counterpart, while macroinstructions, routines, algorithms up to architectures are hierarchical ways to deal with elementary instructions.

This definition extends to the vertical hierarchy of functions the physical approach of Definition 1. Physical variables can be related to each others and located at different levels of a hierarchy. Thus for example the function "to boil" can be described at the level of the mass of water, of molecules of water, of atoms of hydrogen and oxygen, or even further, depending on the functional interest. In general what is crucial here is the possibility to move up and down in the functional description while keeping control of the physical description. At each level we will have different laws of nature operating, and in general it will be possible to relate these laws across layers, since they will have at least some physical variables in common. From this definition we derive two corollaries.

Corollary 3.1 (Levels)

Decomposition of complex functions can be done by following the way in which physical variables are located at different levels of the constitution of the world.

Corollary 3.1 simply states that the bottom up process defined in Definition 3 can be done the other way round, by decomposing complex functions along the lines suggested by the physical variables implied ad various levels. Note that Definition 3 does not imply a reductionist view of science, i.e. the belief that it is possible to explain higher level phenomena by using knowledge related to more elementary level phenomena. It is clearly possible that explanations at various levels are not reducible to each others. As physicists Klein and Lachièze-Rey put it: "Our descriptions of the physical world seem organized in almost independent layers, as if concepts changed their nature as they pile up on top of one another. (…) Complex things cannot be entirely understood on the basis of simple ones, even though the former are nothing more than an aggregation of the latter. Knowledge is not really structured like a set of Russian nesting dolls after all. The fundamental differences between large and small, between simple and complex, demand a change in strategy as the number of degrees of freedom of the system under study increases. Switching from one level of description to the next higher one generally calls for new principles, concepts, and procedures" (Klein and Lachièze-Rey, 1999, p. 108).

Our Definition only states that the functional description can follow the physical description level-by-level, with a degree of understanding and precision which will be a function of the one available at the state of the art of knowledge. From a practical point of view, most functional descriptions are used in a range of size and energy levels for which the explanations available in classical physics, mechanics and chemistry are adequate. As Gerard't Hooft puts it "the fundamental equations underlying the chemical binding forces are completely known". This does not mean it is possible to compute everything, since "unfortunately all calculations starting from these equations are so tremendously complicated that we are forced to use approximation techniques. The accuracy of these mathematical techniques is not always easy to judge, and is often very poor. Even the simplest molecules, such as those of water and alcohol, can often be better studied by doing simple experiments with the substances themselves than by doing *ab initio* calculations starting with our equations" ('t Hooft, 1997, pp. 7-8). Thus it is in general possible to move up and down the scale of the constitution of the world without breaking one's legs. It is honest to admit that functional descriptions of phenomena occurring at scales of matter for which quantum mechanics is needed are not so clear. For example, it is possible that a quantum computer can be eventually manufactured, but its functions are not currently clear at a reasonable detailed level (Lloyd, 2006).

The next two corollaries are more intriguing and have far reaching implications. Corollary 3.2 states that a transformation of a decomposition into another decomposition will be possible, because all decompositions share at least some of the physical variables. More modestly, if two different decompositions share only one physical variable, the transformation will be inevitably partial. Corollary 3.3 states that, since the number of physical variables and of different layers of the physical constitution of the world, is finite, the number of steps implied in any decompositions is finite. And, as the Sub-corollary 3.3.1 makes it clear, there will always be the possibility to stop the top down decomposition by reaching the most elementary level. Since the number of layers in the physical constitution of the world is finite, there must be a layer below which there is no physical description available. This level constitutes the bottom line of physical description.

Let us add, for the sake of completeness, that the most elementary level is defined according to the state of the art of knowledge, leaving room for further scientific advances.

Corollary 3.2 (Transformation of decomposition)
Since multiple decompositions of complex functions are always legitimate, it will in general be possible to identify a transformation, or at least a partial transformation, from a decomposition into another one.

Corollary 3.3 (Finite number of decomposition steps)
Decompositions of complex functions can be done in a finite number of steps.

Sub-Corollary 3.3.1 (Most elementary level)
In moving top down, it will be always possible to reach the most elementary level, one that will not be useful to decompose further.

How can we demonstrate the possibility to link functional descriptions to the physical constitution of the world? We do not offer a formal proof, which will require a deep inquiry into theoretical physics, but rather a constructive proof. In [11]we have shown the Augmented Extended Functional Basis, the base corresponding to a large dictionary (database) including more than 4000 functional elements [12][12]. All these elements are formulated according to Definition 1, i.e. by using physical variables. In order to demonstrate the feasibility of Definition 3 and its corollaries, we started a work of identification of physical effects associated to all physical variables used in functional descriptions. It turns out that it is usually possible to identify them with great precision.

Examples of functional elementary blocks

The question we systematically addressed for each functional description was: is it possible to decompose the function down to an elementary level? In almost all cases this was indeed possible in a quite natural way. Lacking other more original words, we labelled such elementary levels "functional atoms", in analogy with the atomic level in physics. In order to provide the evidence of the functional atoms, some simple examples are supplied below, first using the function set, then the flow set.

1. From the function set:
    - 1.1. Apart from a few very general ones, all functions refer to (a class of) specific objects only: one can **record** only signals, or **to percolate** can only refer to a liquid. These functions have been called by Vermaas [19] "flow restricted" functions. This means that objects of functions are subject to extremely precise definition in physical terms.
    - 1.2. Sometimes the verb determines the initial and the final state of the physical variable as in phase transitions (e.g. **boil** is a transformation of material, necessarily from liquid to gas).
    - 1.3. **Cut**, **drill**, **lathe**, **mill** are all processes for removing material. Hence they belong to the homogeneous cluster whose function is to "remove solid". Yet, they can be distinguished according to the tools used to perform the action. The general description "remove solid" is clearly subject to a precise physical description, while differences in the technology of tooling can also be represented this way.
    - 1.4. In some cases functions differ because of the modality of action. Consider for example the difference between **walk**, **run** and **rush**. In other cases the emphasis is on the variation of the position or of the speed, which allows to distinguish **to move** from **to accelerate**. Position, speed, and their variations are fully describable in physical terms.
    - 1.5. The difference between **touch**, **bump** and **hit** is on the entity of the contact, which is low, medium and high, respectively.
    - 1.6. **Push** and **pull** are equal in modulus and direction but have a different sense (Pull= -Push).
    - 1.7. **Rotate**, **translate** and **twist** differ for the shape of the trajectory they cover, but share the information concerning the motion.
    - 1.8. A process of **polishing** and that of **abrading** can be done by using the same tools (e.g. sand) but differ for the roughness of the achieved surface. Namely they share the fact that multiple tools remove solid from a surface but the size of the abrasive or the duration of the action affect the final result of the action itself.
    - 1.9. The difference may be in the location where the action takes place (i.e. **wrap** vs **stuff**).
    - 1.10. Or, it could be the temporal characteristics of the action that matter (**oscillate** vs **bounce**).
    - 1.11. Finally, **to channelise** belongs to both **to branch** and **to guide**.

In all these examples, it is clear that a precise characterization in physical term is possible, and that differences among functions, usually expressed only in qualitative terms and using natural language, have an extremely precise counterpart in differences in physical descriptions. While this is not a formal proof, it is an encouraging start to formulate all functional expressions in physical language, and to formally identify differences among functions in this way. At the same time, working with large classes of examples allows to say that, in general, the problem of allocating functional descriptions at different levels of the constitution of matter is tractable. In almost all cases there is no difficulty in moving up and down the scale of physical variables and identifying the appropriate level for any decomposition of complex functions. Corollary 3.3 is indeed realistic. Let us turn now to the flow set.

2. From the flow set:
    - 2.1. The notion of **plasma** can be "atomically" decomposed in ions of gas with high energy and high temperature.
    - 2.2. The case of **mixtures** is interesting. They not only can be decomposed in two or more primary flows (solid, liquid, gas) but also more information can be added on physical features of the mixture: whether homogeneous or not, ordered or chaotic, viscous or fluid, etc.
    - 2.3. With **sound/ultrasound** wave, information on energy and on signal has to be simultaneously managed. We propose to consider sound as an atom with two fathers (energy and signal).
    - 2.4. A similar case happens for **alloys,** which can be considered simultaneously solid (at the first sight) and mixture (if we consider the inner nature of the solid).
    - 2.5. **Electromagnetic** waves can transmit both energy and signal, so they should be characterized simultaneously in both dimensions.
    - 2.6. **X rays** are another interesting case. Their overall function (i.e. generate a printable image) can be decomposed with respect to light energy, initially void of signal content, but eventually bringing a signal via the physical interaction with the material.
    - 2.7. **Liquids** and **gases** are classified in functional bases (see for example the Reconciled Functional Basis) according to homogeneity, compressibility, or elasticity, all physical variables that can be described down to the elementary property.

Again, no difficulties arise in the effort to formulate flows into clearly defined physical descriptions, nor in the effort to find a rationale in decomposing complex functions.
Previous examples are particularly easy since they differ just in an element. More often, also very known and common functions differ from another (similar at the first sight) in more than one element. Let us show the example of **boil** and **evaporate**. To boil is the process of transforming water into vapour, which happens *in the mass of the liquid* and in an *irreversible* way. To evaporate shares the first part of the functional definition but differs from boil because it happens *at the liquid surface* in a *reversible* manner. In this case differences are more subtle and complex. But, again, a careful analysis in physical terms gives full justice of all differences. Let us finally refer to the early work by Pahl and Beitz [6]. Interestingly, we find clear evidence of an effort to make systematic use of physical description, although the final framework paid a tribute to the need for comprehensiveness and introduced many loose formulations.

3. Other examples coming from Pahl and Beitz's work are as follows:
    - 3.1. Pahl and Beitz felt the necessity of exploring the flows by adding some information to the simple state of the material (i.e. solid, liquid, gaseous). Actually, they distinguish materials by introducing their **behaviour** (rigid, elastic, plastic, viscous, hard, …) and their **form**, if any (solid body, grains, powder, dust). These can be considered as proto-functional atoms.
    - 3.2. In the description of **cynematic motion** they introduce the categories of type of motion (translational, rotational), nature of motion (regular, irregular), direction, magnitude (velocity etc), number (single, multiple). Clearly all these categories have a natural counterpart in physical descriptions, in most cases via equations.
    - 3.3. Pahl and Beitz realize the role of **opposition relations**. For example they state that "to seal liquid" is equivalent to "do not channel liquid". This intuition, further developed by Hundal with the notion of antonymy, has a strong physical characterization.
    - 3.4. Another interesting point is the distinction between the energy that is ready for use or in use, as implicit in forces and physical effects, and the energy that is stored and can be potentially used in certain conditions. The types of energy involved are the same: rotational, kinetic energy is underlying both the centrifugal force ready to be used and the flywheel effect. Again they separate capacitive energy from capacitor stored energy. In our language, Type of energy and Conditions for use of energy are elementary (or atomic) functional descriptions.

We then find evidence in the early work of pioneers of functional analysis of a keen understanding of the importance of physical descriptions, with *in nuce* an articulation across levels of reality. Pahl and Beitz were in fact ready to introduce, along with verb + object descriptions, a large number of other elements, such as geometry, cost, safety, etc. All these elements confirm their intuition that the context of functions is relevant, so that many elements must concur in a fully adequate functional description.

BREAKING WITH TREE-LIKE REPRESENTATIONS

According to the quotation from Lewontin, a strong limitation of current functional approaches is that they do not permit functions to be part of different hierarchies. In other words, functions are part of tree-like structures, in which each branch has just one node above it. Tree-like structures are adequate to represent perfectly partitioned graphs, as it happens in taxonomies. But they are deeply inadequate if functions have not only one, but more higher level functions. This can be expressed as multiple inheritance: a lower level function has two fathers, i.e. two higher level functions of totally different nature. Indeed, as we have shown in [10], existing functional bases, including the authoritative Reconciled Functional Basis, are logically contradictory, because they claim to follow a tree-like, partitioned hierarchy, representation, while in fact there are several cases of multiple inheritance.

We have suggested to give away the tree-like representation and to adopt a graph-theoretic representation that explicitly allows for multiple inheritance. Such representation fully meets Lewontin's call for horizontal causal pathways. In doing so, we are aware of the risks. Tree-like representations are more intuitive than graph representations. They also allow for better grasping of elements in graphical forms. However, this is in part due to the limitations of current functional models, which do not allow navigation in large maps. We are fully convinced that new technologies for navigation in graphical representations, supported by graph structures, will allow designers to move naturally into large functional maps.

Definition 4 (Graph structure)

Functions can be represented as nodes linked to other nodes vertically and horizontally.
Vertical relations describe different levels of abstraction, at the top, and decomposition, at the bottom (father-son hierarchical relations). Horizontal ones describe functions that are synonyms or antonyms.

Corollary 4.1 (Multiple inheritance)
It is possible for any given function to have two or more fathers in vertical functional decompositions.

The above corollary may be obtained also from Definition 1 that states that concurring actions may be implied in functional descriptions. For example **to convey** implies both a transmission and a guidance; moreover it implies a selective blockage and sometimes a branching. Multiple inheritance can be also defined by stating that a function C has the same atoms of both A and B (where A and B have no atoms in common). Other examples are the function **insulate** (which is a separation of two flows obtained by the addition of insulating material, all with the purpose to protect one of the separated flows) or the function **fasten** (that embeds the concepts of both blocking some degrees of freedom and supporting / securing the object fastened).

Corollary 4.2 (Bridging)
When a function is subject to multiple inheritance, there will be a bridge in the semantic content of clusters of functions that would otherwise be disconnected.

In a graph representation of the functional dictionary several clusters of homogeneous synonyms emerge, but some of the elements of a cluster can be connected (via affinity relationship) with other clusters of homogeneous synonyms themselves. Those elements bridging the two clusters are functions whose atoms belong to both clusters.

For example the function **cover** is connected to both the **pack** cluster of synonyms and the **coat** cluster. In addition, *cover* is the representative of its own group of homogeneous synonyms. Clearly the common ground for the three clusters is the addition of a layer over the system at hand. Even if the layer is added for different purposes in the three different semantic areas, the basic action of adding such layer must be present in all of them. Other atomic elements will take care of describing the differences between the three groups. Another example is given by the three groups represented by **capture**, **enter** and **contain**. Capture is actually connected to enter and contain groups too, thus creating an affinity bridge between them. Analysing the linking function we have an indication of the common atoms between the three clusters: those expressing the import of the object inside the system. Our representation not only permits multiple inheritance, but fully exploits the semantic information associated to it and to the bridging effect. By identifying multiple inheritance and bridges among clusters of functions, in fact, it is possible to apply heuristic methods to generate new information.

The combination of a physical approach with a graph-theoretic representation also implies a novel definition of the issue of affinity between functions, an issue with many practical implications that has not found so far any satisfactory solution. Let us then derive a formal corollary on this issue.

Corollary 4.3 (Affinity)
Two perfect functional synonyms have the same decomposition. The degree of affinity is proportional to the number of functional atoms shared by two or more functions.

According to this corollary, the number of perfect synonyms in most functional bases reduced drastically. Just a smaller subset of functions can be described by using different terms. Consider the following examples*:* bound vs delimit / contain vs include / adapt vs adjust / bend, bow, curve, fold, hump (all synonyms) / delete, cancel, erase / convey vs channelise. Nearly all the elements of a homogeneous cluster are nearly synonyms but they differ for some atoms (the number of atoms they differ is a measure of the cluster kurtosis). For example, in phase transitions (clustered around **convert**): there is a difference in the regime of the phase transition between **boil** and **evaporate**, whether irreversible and chaotic or reversible and smooth. In turn, the couple **evaporate/ boil** differs from **liquefy** due to the entropic direction, and differs even more greatly from **solidify** and others phase transitions due to differences in initial and final states.

Another example is the cluster represented by absorb. Its meaning may be to remove the liquid from the system, or to wet the absorbing object. There will functional atoms shared by all meanings (i.e. import a liquid) and functional atoms differentiated. Furthermore, in the same cluster there may be differences according to the object absorbed, for example a liquid or a radiation.

In all these examples there are affinity relations, whose importance is difficult to establish via a qualitative, linguistic investigation. By forcing their functional descriptions towards physical variables, on the contrary, it is possible to identify with great clarity the degree of affinity of functions. The number of common functional information (atoms) between a function A and the affine function B can be used to create a measure of the affinity level.

CONCLUSIONS

In this paper we have offered constructive proofs of the great potential of a new approach to functional analysis, based on three main options: adopting a physical description, possibly associated to equations; decomposing complex functions by following the internal constitution of physical world, and using a graph representation allowing for multiple inheritance and measurement of affinity. Such approach is backed by the construction of a large highly structured functional basis, including more than 4000 functional expressions. While a formal proof of the superiority of this approach with respect to the existing ones is still to be developed, we have shown a large number of highly informative examples. Many research directions are opened by such bold moves.

First, we will fully provide a physical atomic description for all 4000 functions already in the Extended Functional Basis. Computer power and text mining software will help the research in understanding and discovering the functional atoms on which all functions are based on. A first estimate performed by the authors is in the order of about 150 high frequency functional atoms plus a plethora of rare ones (mainly due to the different tools used in manufacturing operations).

Second, we aim at covering all 4000 functions already available with clear identification of physical effects, at various scales of complexity, and using these results to demonstrate that functional decompositions can be made rigorous. This is crucial to develop a formal language which can fully translate the world of technical specifications with the world of descriptions, usually formulated in natural language, of value of artefacts to stakeholders, such as users or buyers. It is common observation that it is extremely difficult, within and across organizations, to establish a fruitful dialogue between those that build representations of value to stakeholders (who talk in terms of values, ends, needs) and technical experts (who only know product attributes, behaviours, specifications). Our approach is able to translate stakeholders representations in natural language into a formal description that can be managed by engineers and has already a number of applications in company contexts.

Third, once the coverage of atomic descriptions will be complete, we will introduce distance measures in order to compute a number of topological properties. Using the functional atomic decomposition to be implemented in a database it is possible to measure the functional space and then the degree of similarity between two products, systems, patents or procedures, or classes of them.

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REFERENCES

The list of references is limited to the engineering design theory. Additional references quoted in the paper are available at http://bonaccorsi.blogspot.com/paper/ICED09

[1] Miles L.D. *Technique of Value Analysis and Engineering*, 1972 (Mc Grow-Hill Book Company)

[2] G. Boothroyd, Product Design For Assembly, 1986 (Boothroyd Dewhurst Inc., Wakefield,).

[3] Franceschini F. *Advanced Quality Function Deployment*, 2002 (CRC Press, Boca Roca, Florida).

[4] Altshuller, G., “Creativity As An Exact Science”, (New York, Gordon And Breach, 1988).

[5] Braglia M., Fantoni G. and Frosolini M. The House of Reliability, 2006, *International Journal of Quality and Reliability management (IJQRM)*.

[6] Pahl G. and Beitz W *Engineering Design: A Systematic Approach* 1984(Design Council, London)

[7] Collins, J., Hagan, B., and Bratt, H. (1976), The Failure-Experience Matrix - a Useful Design Tool, *Transactions of the ASME, Series B, Journal of Engineering in Industry*, 98:1074-1079.

[8] Szykman S., Racz J.W., Sriram R.D. The Representation of Function in Computer-based Design, *ASME Design Engineering Technical Conferences, DETC99/DTM-8742*, 1999, Las Vegas, NV.

[9] Hirtz J., Stone R., McAdams D., Szykman S. and Wood K., A Functional Basis for Engineering Design: Reconciling and Evolving Previous Efforts, *Research in Engineering Design*, 2002, 13, 65-82.

[10] Apreda, R., Bonaccorsi, A., and Fantoni, G., “Rethinking Functional Analysis”, 2008, submitted to *Research in Engineering Design.* available at http://bonaccorsi.blogspot.com/paper/ICED09

[11] Apreda, R., Bonaccorsi, A., Fantoni, G., “Functional Vector Space”, 2009, submitted to *ICED09*. and available at http://bonaccorsi.blogspot.com/paper/ICED09

[12] Bonaccorsi, A., Fantoni, G., (2007) Expanding the functional ontology in conceptual design. *International Conference on Engineering Design*, ICED 2007, Paris, 28 - 31 August 2007.

[13] Gero J.S. Design prototypes: a knowledge representation schema for design. AI Magazine, 1990, 11(4), 26-36.

[14] Gero JS, Kannengiesser U (2002) The Situated Function-Behaviour-Structure Framework, Artificial Intelligence in Design ‘02. Kluwer, Dordrecht, pp 89–104

[15] Bonaccorsi A. Grammars of creation. Mapping search strategies for radical innovation, 2006, *Innovation Pressure Conference*.

[16] Mills J. J. and Iyer, G. (2007) Using Domains to Constrain Design Variables. *International Conference on Engineering Design*, ICED 2007, Paris, 28 - 31 August 2007.

[17] Nagel, R. L., Vucovich, J. P., Stone, R. B. and McAdams, D. A. “Signal Flow Grammar, From the Functional Basis”, *International Conference on Engineering Design*, ICED’07 Paris, France

[18] Sridharan, P. and Campbell, M.l. (2004) A Grammar for Functional Structures. Design *Engineering Technical Conferences* and *Computers and Information in Engineering Conference* (ASME, Salt Lake City, Utah, 2004).

[19] Vermaas P., The Functional Modelling Account of Stone and Wood: Some Critical Remarks. *International Conference on Engineering Design*, ICED 2007, Paris, 28 - 31 August 2007.

[20] Dorst, K., Vermaas, P. E., John Gero’s Function-Behaviour-Structure model of designing: a critical analysis, *Research in Engineering Design*, (2005) 16: 17–26.

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