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A SYSTEMATIC APPROACH FOR COST OPTIMAL TOLERANCE DESIGN

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ABSTRACT
This paper presents a new method and a prototype system for cost optimal tolerance design in mechanical assemblies. The system overcomes inefficiencies and/or handicaps met in current methods and tools that require expert cost-tolerance input referenced to the participating in the assembly chain dimensions or the availability of case-driven experimental cost-tolerance data. This is accomplished by introducing the concept of the Tolerance Element (TE), a machining process related geometric entity with attributes associated with the accuracy cost and identifiable in conformance with the standard industrial understanding. It becomes thus possible to assign cost optimum and rational component tolerances through an algorithmic mode that can be directly integrated in a CAD environment. Within this -based on the TE approach- frame, machining capability and cost per TE-class of a particular machine shop are appropriately recorded and processed and cost-tolerance functions automatically established and stored in the system database to be used for tolerance optimization. The latter once created, needs then updating only when changes take place in the machine shop resources and/or expertise. Tolerance allocation can be thus achieved with minimum manufacturing cost within the machine shop accuracy getting capability in an effort and time efficient way through realistic and machine shop focused cost-tolerance data. An application example demonstrates the method and its positive evaluation in comparison with alternative problem solutions.

Keywords: Tolerancing, Tolerance allocation, Cost-Tolerance Function, Optimum Tolerancing, Tolerances Chains

1 INTRODUCTION

Within the frame of further development of CAD tools emphasis is given to computer-aided tolerancing techniques for cost optimal allocation of tolerances in mechanical assemblies [1-4]. In engineering design as optimal tolerancing is meant the assignment of tolerances to the components of a mechanical assembly in terms not only of functionality but also of minimum manufacturing cost. The functional performance of an assembly, as a result of the deviations of its critical dimensions, is decisively affected by the component tolerances according to the tolerance chains. Tight tolerances generally result to high dimensional accuracy, quality and modularity of the product. High accuracy is mainly based, on the other hand, on machining processes and always imposes additional effort hence higher manufacturing costs. Given that accuracy costs constitute a vital issue in the industrial production, various cost-tolerance relationships have been proposed and tried to this date, [3,5-8]. Critical characteristic of these relationships however, is that their exponents and coefficients are obtained experimentally under particular each time experimental conditions and cannot therefore be broadly used.

CAD software for cost optimal tolerance allocation does not make use of such case-driven cost-tolerance functions as they may well not be representative of the machine shop where the components will be manufactured. Instead, they require from the user to feed the system with cost-tolerance data based on his own judgement and cost-tolerance knowledge. The procedure is, apparently, not systematic and subjective to the way the particular user understands the accuracy cost of a machining tolerance. It can also be time-consuming in case the assembly has a large number of member-tolerances. Considerable dependency of the produced results on user input, misinterpretations, mistakes and unnecessary repetitions are unavoidable.

In this paper a prototype system for cost optimal tolerance design in mechanical assemblies is presented. The system overcomes inefficiencies met in existing CAD tools that require from the user expert cost-tolerance input or the availability of case-driven experimental cost-tolerance data. The developed methodology introduces the concept of the Tolerance Elements that are geometric entities with attributes associated with the accuracy cost of the specific machining environment where the components will be manufactured. Once established for this environment, the system automatically creates and makes use of appropriate cost-tolerance functions for the assembly chain members under consideration. Cost optimal tolerance design can thus be achieved in a systematic, cost and time efficient way by means of realistic and machine shop focused cost-tolerance data. The benefits of this system are demonstrated and discussed through an application example.

2 THE METHOD

2.1 Tolerance Elements

Accuracy cost is the cost required to produce a given dimension within its specified tolerance limits. It depends, apparently, on the processes and resources needed for the part production. Given the workpiece material and tolerances, the part geometrical characteristics such as size, shape, feature details, internal surfaces, are taken into consideration for planning the machining operations, programming the machine tools, specifying fixtures, etc. These geometrical characteristics have thus a direct impact on the machining cost of the required accuracy by determining, indirectly, its magnitude [9]. A Tolerance Element (TE) is defined as a 3D form feature of particular shape, size and tolerance. It incorporate attributes associated with its shape, size, position, the presence of additional feature details and the ratio of the principal dimensions of the part to which it belongs. To each TE corresponds one cost-tolerance function that stands for its accuracy cost and is directly related with the particular machine shop where the TE will be produced (machine tools, inspection equipment, supporting facilities, expertise).

Tolerance Elements are classified through a five level class hierarchy system, Figure 1. Class level attributes are all machining process related, generic and straightforwardly identifiable in conformance with the existing industrial understanding.

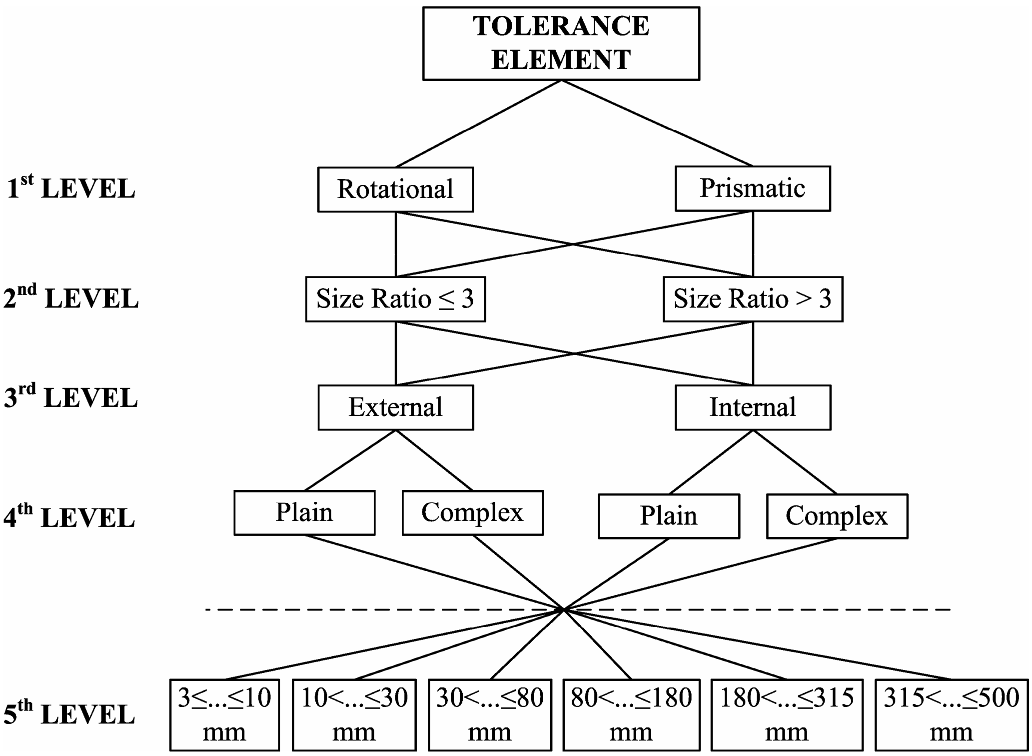


Figure 1. TE classification

In first level, TEs are classified according to the basic geometry of the part to which they belong, i.e. *rotational* TEs and *prismatic* TEs. Rotational TEs belong to rotational parts manufactured mainly by turning and boring, while prismatic TEs belong to prismatic parts mainly manufactured by milling.
In second level, TEs are classified according to the size ratio of the principal dimensions of the part to which they belong, considering the required resources for part machining and/or the elastic deformations due to cutting forces or part own weight. In this way TEs are classified as *short* [L/D ≤3] and *long* [L/D >3] TEs, following a typical way of classification [10-11]. For a rotational part, D represents its largest diameter, L the part length. For a prismatic part L represents its largest dimension and D the largest one in the direction of the two remaining Cartesian axes.
In third level TEs are classified to *external* and *internal* ones as the achievement of tight tolerances in internal TEs usually results to higher accuracy cost.
The fourth TE classification level distinguishes between *plain* and *complex* TEs depending on the absence or presence of additional feature details. Such details include grooves, wedges, ribs, threads or notches. They do not change the principal TE geometry but they indirectly contribute to the increase of the accuracy cost.
In the final fifth level, the involvement of the TE size to the accuracy cost is considered. TEs are classified, according to the nominal size of their participating in the chain dimension, into six groups by integrating two sequential ISO 286-1 size ranges. The five-digit TE classification code is shown in Table 1. Figure 2 illustrates a rotational part with rotational TEs and their codes.

Table 1. TE coding scheme

DIGIT 1		**DIGIT 2**		**DIGIT 3**		**DIGIT 4**	
1	Rotational TE	1	Short TE	1	External TE	1	Plain TE
0	Prismatic TE	0	Long TE	0	Internal TE	0	Complex TE

DIGIT 5	
1	3≤ Dimension ≤10 mm
2	10< Dimension ≤30 mm
3	30< Dimension ≤80 mm
4	80< Dimension ≤180 mm
5	180< Dimension ≤315 mm
6	315< Dimension ≤500 mm

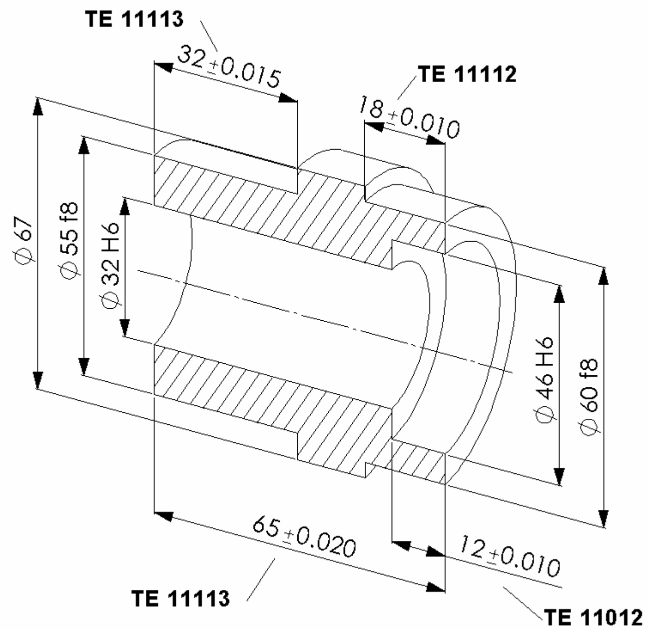


Figure 2. Rotational TEs (assembly chain in the axis direction)

2.2 Modelling of cost-tolerance functions

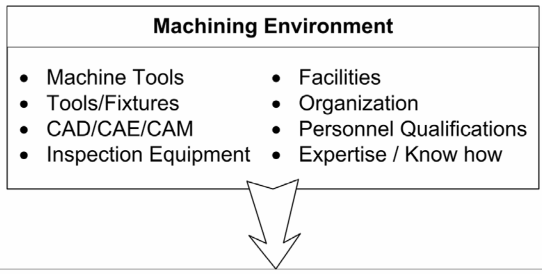
Based on the TE-method the actual machining accuracy capabilities and the relative cost per TE-class of a particular machine shop are recorded through the Database Feedback Form (DFF) of Figure 3. The latter includes the accuracy cost for all the 2×2×2×2×6=96 TE-classes in the size range 3-500 mm and tolerances range IT6-IT10. The relative cost concept represents, on a comparative basis, the exponential increase of costs required for expertise, effort and resources as the tolerances become tighter, [3,6,8,11]. The 10-grade relative cost scale is shown in Table 2. A DFF is filled once, when the system is commissioned, by the expert engineers of the machine shop where the assembly components will be manufactured. It can then be updated each time changes occur in the shop machines, facilities and/or expertise.

Processing of the database data is performed per TE-class through the least-squares approximation in order to construct the cost-tolerance relationship of the power function type [1,7],

$$C(t) = A + B / t^k \qquad (1)$$

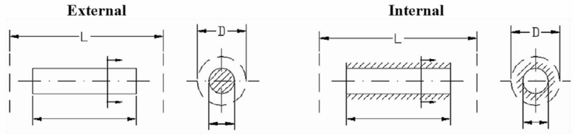
In equation (1) C(t) is the relative cost for the production of the machining tolerance ±t and A, B, k are constants. Constant A represents the cost for producing the TE dimension with highly relaxed accuracy. Within the frame of this work this cost component is constrained to the relative cost of IT10. The second part of equation (1) constitutes, therefore, the accuracy cost for the production of the TE-dimension with increased accuracy.

Cost-tolerance functions established with the described way for all the TE-classes are stored in the system database and become thus available for machine shop focused cost optimum tolerancing.



Database Feedback Form

Rotational Elements



TOLERANCE GRADE IT6

a/a	Reference Dimension (mm)	Tolerance Zone T (μm)	External				Internal			
			Plain		Complex		Plain		Complex	
			L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D
1	3-10	8 – 9								
2	10-30	11 – 13								
3	30-80	16 – 19								
4	80-180	22 – 25								
5	180-315	29 – 32								
6	315-500	36 – 40								

TOLERANCE GRADE IT7

a/a	Reference Dimension (mm)	Tolerance Zone T (μm)	External				Internal			
			Plain		Complex		Plain		Complex	
			L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D
1	3-10	12 - 15								
2	10-30	18 - 21								
3	30-80	25 - 30								
4	80-180	35 - 40								
5	180-315	46 - 52								
6	315-500	57 - 63								

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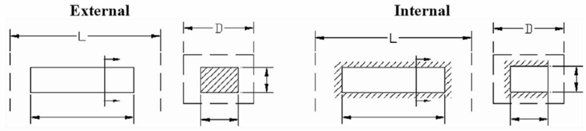
TOLERANCE GRADE IT10

a/a	Reference Dimension (mm)	Tolerance Zone T (μm)	External				Internal			
			Plain		Complex		Plain		Complex	
			L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D
1	3-10	48 – 58								
2	10-30	70 – 84								
3	30-80	100 – 120								
4	80-180	140 – 160								
5	180-315	185 – 210								
6	315-500	230 – 250								

Prismatic Elements



TOLERANCE GRADE IT6

a/a	Reference Dimension (mm)	Tolerance Zone T (μm)	External				Internal			
			Plain		Complex		Plain		Complex	
			L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D
1	3-10	8 – 9								
2	10-30	11 – 13								
3	30-80	16 – 19								
4	80-180	22 – 25								
5	180-315	29 – 32								
6	315-500	36 – 40								

TOLERANCE GRADE IT7

a/a	Reference Dimension (mm)	Tolerance Zone T (μm)	External				Internal			
			Plain		Complex		Plain		Complex	
			L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D
1	3-10	12 - 15								
2	10-30	18 - 21								
3	30-80	25 - 30								
4	80-180	35 - 40								
5	180-315	46 - 52								
6	315-500	57 - 63								

………………………………

TOLERANCE GRADE IT10

a/a	Reference Dimension (mm)	Tolerance Zone T (μm)	External				Internal			
			Plain		Complex		Plain		Complex	
			L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D	L≤3D	L>3D
1	3-10	48 – 58								
2	10-30	70 – 84								
3	30-80	100 – 120								
4	80-180	140 – 160								
5	180-315	185 – 210								
6	315-500	230 – 250								

Figure 3. Database Feedback Form

Table 2. Relative cost scale

Machining	Relative Cost
Unfeasible	10
Extremely difficult	9
Quite difficult	8
Difficult	7
Feasible with some effort	6
Feasible	5
Easily feasible	4
Easy	3
Quite easy	2
Extremely easy	1

3 TOLERANCE DESIGN

In a n-member dimensional chain the tolerances of the individual dimensions D_i, i=1,2,…,n, control the variation of a critical end-dimension D_0, according to the chain,

$$D_0 = f(D_1, D_2, ..., D_n) \tag{2}$$

where f (D_i) can be either a linear or nonlinear function. To ensure that the end-dimension will be kept within its specified tolerance zone, two approaches are usually employed for tolerance allocation. The worst-case constrain, that provides for 100% interchangeability, results usually to tight tolerances and hence to high accuracy costs,

$$\frac{\partial f}{\partial D_1} t_1 + \frac{\partial f}{\partial D_2} t_2 + ... + \frac{\partial f}{\partial D_n} t_n \le t_0 \tag{3}$$

In case that a certain rejection percentage can be considered as acceptable, then the root-sum-squares constrain that implies the normal distribution for the dimensional deviations is used instead,

$$\sqrt{\left(\frac{\partial f}{\partial D_1}\right)^2 t_1^2 + \left(\frac{\partial f}{\partial D_2}\right)^2 t_2^2 + ... + \left(\frac{\partial f}{\partial D_n}\right)^2 t_n^2} \le t_0 \tag{4}$$

In above equations t_0 and t_i are the tolerances of the end-dimension D_0 and the dimension D_i respectively.

An additional constrain that arises from the capability of the process and machine tools to be used is,

$$t_i \ge T_m \quad m = X, Y, Z \tag{5}$$

with T_m representing the available accuracies along the machining axes X, Y, Z.

Cost optimum tolerancing of the assembly chain dimensions can thus be pursued through the minimization of the nonlinear objective function,

$$C_{total}(t_1,..,t_n) = \sum_{i=1}^{n} C_i(t_i) = \sum_{i=1}^{n} \left[A_i + B_i / t_i^{k_i}\right] \rightarrow \min \quad , \quad \sum_{i=1}^{n} B_i / t_i^{k_i} \rightarrow \min \tag{6}$$

subject to the constrains (3) or (4) and (5).

4 METHOD IMPLEMENTATION

The prototype software has been developed using Microsoft Excel and Matlab tools. It consists of two major modules, namely the database module and the tolerance allocation module. For the development of the database module the add-in Excel Link is used that allows for data exchange between the two environments. An expert engineer of the machine shop fills the Database Feedback Form at the system commissioning stage and sets the current machine shop accuracy capability. The database module runs only at this stage and any time since DFF needs updating. Cost-tolerance functions for the TE-classes are thus automatically computed and stored in the database.

The second module that deals with tolerance allocation consists of three submodules: (i) module for the requirements definition, (ii) module for TE and cost-tolerance function identification, and (iii) module providing for the optimum solution through the minimization of the automatically established objective function. A user-friendly interface has been developed to provide the user with an interactive windows environment for easy and efficient system operation. Sequential Quadratic Programming (SQP) algorithm of the Matlab Optimization Toolbox for constrained nonlinear optimization is used. Starting point of the algorithm is taken the tolerance grade IT10. The flow charts of the database and the tolerance allocation modules are shown in Figures 4 and 5 respectively.

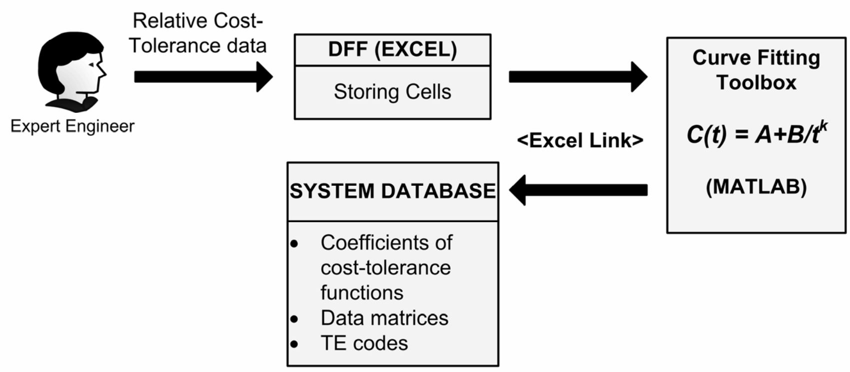


Figure 4. Database module

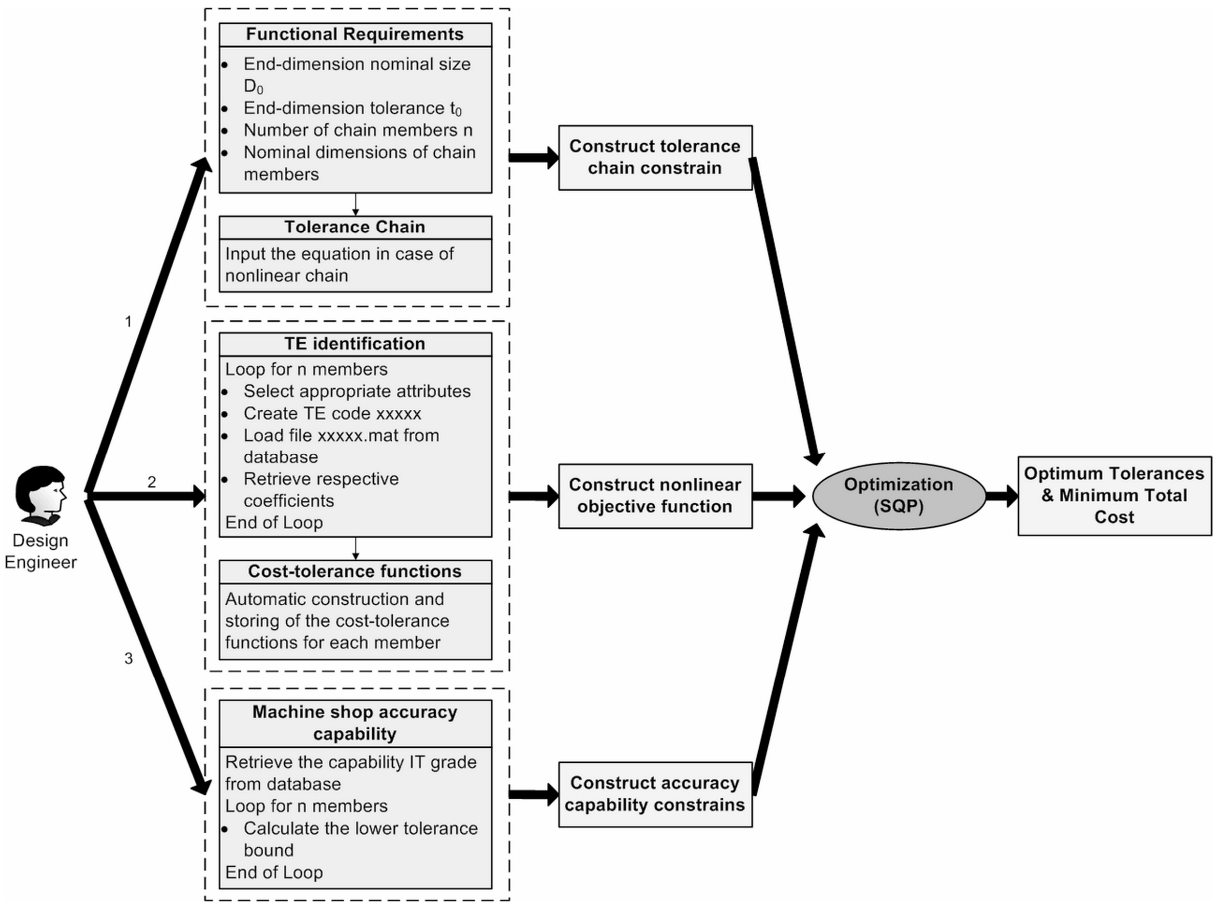


Figure 5. Tolerance allocation module

5 APPLICATION EXAMPLE AND DISCUSSION

In the assembly of components A-B-C of Figure 6 below the dimension $D_4 = 75 \pm 0.25$mm is controlled through the two dimensional chains (D1,D2,D3,D4,D5,D6,D7,D8), (D8,D9,D10,D11). Worst case cost optimum tolerancing is required for all the chain dimensions. The machine shop where the assembly components will be manufactured has an IT6 best capability and its DFF processed and results stored.

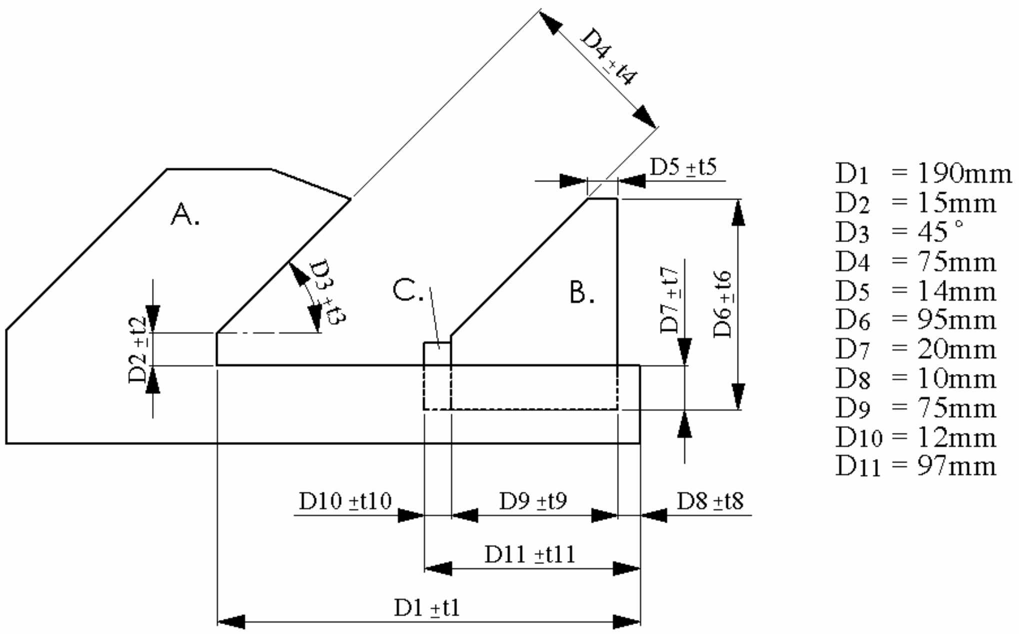


Figure 6. Application example

Chain equations involved are,

$$D_4 = \left[D_1 - \left(D_6 - D_7 - D_2\right)\cdot \cot D_3 - D_5 - D_8\right]\cdot \sin D_3 \tag{7}$$

$$D_8 = D_{11} - D_{10} - D_9 \tag{8}$$

Case TEs and cost-tolerance functions retrieved by the algorithm from the database are shown in Table 3. The total cost model was constructed in accordance with the relationships (3), (5) and (6) and the optimization process finally produced the results quoted in Table 4. For system performance evaluation are also given, in the same table, the results produced by two alternative approaches, a conventional (not optimized) mode that applies same IT grade, here IT8, for all the chain dimensions starting from the highest IT that satisfies the constrains and a commercially available tolerancing tool. Users A and B of the commercial tool followed User Instructions and worked independently using their own cost-tolerance knowledge to drive the tool and produce the results. The total accuracy cost of all three approaches was calculated using the cost-tolerance functions of the present method, Table 3.

Table 3. Cost – tolerance functions of the application example

Dimension	TE	Cost – tolerance function
D_1	01115	$C_1(t_1) = 3 + 0.06008/\left(t_1^{0.8983}\right)$
D_2	01112	$C_2(t_2) = 5 + 0.02282/\left(t_2^{0.9004}\right)$
D_3	01114	$C_3(t_3) = 3 + 0.07138/\left(t_3^{0.9149}\right)$
D_5	01112	$C_5(t_5) = 5 + 0.02282/\left(t_5^{0.9004}\right)$
D_6	01114	$C_6(t_6) = 3 + 0.07138/\left(t_6^{0.9149}\right)$
D_7	01012	$C_7(t_7) = 7 + 0.01022/\left(t_7^{0.9309}\right)$

D_9	01113	$C_9(t_9) = 4 + 0.03349 / \left(t_9^{0.9535}\right)$
D_{10}	01112	$C_{10}(t_{10}) = 5 + 0.02282 / \left(t_{10}^{0.9004}\right)$
D_{11}	01014	$C_{11}(t_{11}) = 7 + 0.02023 / \left(t_{11}^{0.9141}\right)$

Table 4. Application example results

Dimension	IT8	Tolerancing Tool		Present system
		User A	User B	
D_1 = 190mm	± 0.036	± 0.095	± 0.071	± 0.044
D_2 = 15 mm	± 0.013	± 0.011	± 0.018	± 0.026
D_3 = 45˚	± 0.027˚	± 0.010˚	± 0.015˚	± 0.025˚
D_5 = 14 mm	± 0.013	± 0.010	± 0.026	± 0.027
D_6 = 95 mm	± 0.027	± 0.010	± 0.069	± 0.050
D_7 = 20 mm	± 0.016	± 0.087	± 0.069	± 0.019
D_9 = 75 mm	± 0.023	± 0.021	± 0.017	± 0.037
D_{10} = 12 mm	± 0.013	± 0.014	± 0.010	± 0.027
D_{11} = 97 mm	± 0.027	± 0.065	± 0.020	± 0.026
Accuracy cost:	10.747	15.655	10.178	7.730

As it can be seen from the results, minimum total cost was achieved by the presented system. The system leads to the same tolerances whatever is the IT grade the algorithm is asked to start the accuracy cost optimization. That does not happen with the commercial tool whose results are also strongly user input dependent. Time needed to operate such a system is also much longer than that of the present method. The conventional approach, on the other hand, although very simple cannot guarantee an optimum solution as it does not take into consideration cost influencing parameters other than the dimensional size. Similar results from other case studies confirm these observations.

6 CONCLUSIONS

Assignment of proper tolerances to the components of a mechanical product is of major importance for the product quality, functionality, modularity and manufacturing cost. This paper introduced the Tolerance Element concept and presented the development of a prototype system for the establishment and processing of rational and machine shop focused cost-tolerance information. The new methodology overcomes inefficiencies and handicaps of the currently available tolerancing tools and can be easily integrated into a CAD environment. The system database once created needs updating only when changes occur in the machine shop resources and/or expertise. The developed system was used for cost optimal tolerance design in mechanical assemblies and produced realistic results in a small fraction of time otherwise required by alternative -computer aided or not- problem solutions.

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