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# A NEW APPROACH TO MULTI-CRITERIA MATERIAL SELECTION IN ENGINEERING DESIGN

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## Abstract

The selection of a material to meet given design requirements generally requires that a compromise be struck between several, usually conflicting, objectives. The problem is complicated because the decision-space is large, it is discrete rather than continuous, and the relative value to be placed on each objective is imprecisely known. Here we explore the ways in which multi-objective optimisation methods can be adapted to address this problem. We find that trade-off surfaces give a way of visualizing the alternative compromises, and that "utility" functions (or "value" functions) identify the part of the surface on which optimal solutions lie. Implementing this for two objectives is straightforward, but doing so for more then two requires new visualization tools. Here we develop a tool and illustrate its use.

*Keywords: multi-objective optimisation, innovative methods, engineering diagrams*

# 1. Introduction

Real-life decision-making frequently requires that a compromise be reached between conflicting objectives. The compromises required to strike a balance between the performance and the cost of a car, or between health and the pleasure of eating rich foods, or between wealth and quality of life, are familiar ones. Similar conflicts arise in the choice of materials. The objective in choosing a material is to optimise a number of *metrics of performance* in the product in which it is used. Common among these metrics are *cost, mass, volume, power-to-weight ratio* and *energy density*, but there are many more. Conflicts arise because the choice that optimises one metric will not, in general, do the same for the others; then the best choice is a compromise, optimising none but pushing all as close to their optima as their interdependence allows. This paper concerns multi-objective optimisation of material choice. It extends established methods for multi-objective optimisation [1-6] and for material selection [7, 8]. The methods are equally applicable to material selection, and to the inverse problem of identifying promising applications for new materials.

# 2. Optimised material selection

## 2.1 Multi-objective optimisation and trade-off surfaces

Any engineering component has one or more functions: to support a load, to contain a pressure, to transmit heat, and so forth. In designing the component, the designer has an objective: to make it as cheap as possible, perhaps, or as light, or as safe, or some combination of these. This must be achieved subject to constraints: that certain dimensions are fixed, that the component must carry the given mechanical, thermal and electrical loads without failure, that it can survive for its design-life in a given environment, and many more.

When there are two or more objectives, solutions rarely exist that optimise all at once. Economic lightweight design is an example: the best solution (including material choice) is the one that minimises both the weight and the cost. The example nicely illustrates the difficulties: the objectives are measured in different units (here, kg and $) and they are in conflict, meaning that any improvement in one usually results in deterioration in the other. The situation is illustrated for two objectives by Figure 1(a) in which one performance metric, $P_2$, is plotted against another, $P_1$. Minima are sought for both. Each bubble describes a solution. The solutions that minimise $P_1$ do not minimise $P_2$, and vice versa. Some solutions, such as X and Y, are far from optimal – other solutions exist with lower values of both $P_1$ and $P_2$. Solutions like X and Y are said to be *dominated* by others. Solutions like those at A, B and C have the characteristic that no other solutions exists with lower values of both $P_1$ and $P_2$. These are said to be *non-dominated* solutions. The line or surface on which they lie is called the non-dominated or optimal *trade-off surface*. The values of $P_1$ and $P_2$ corresponding to the non-dominated set of solutions are called the *Pareto set* [1, 3, 9].

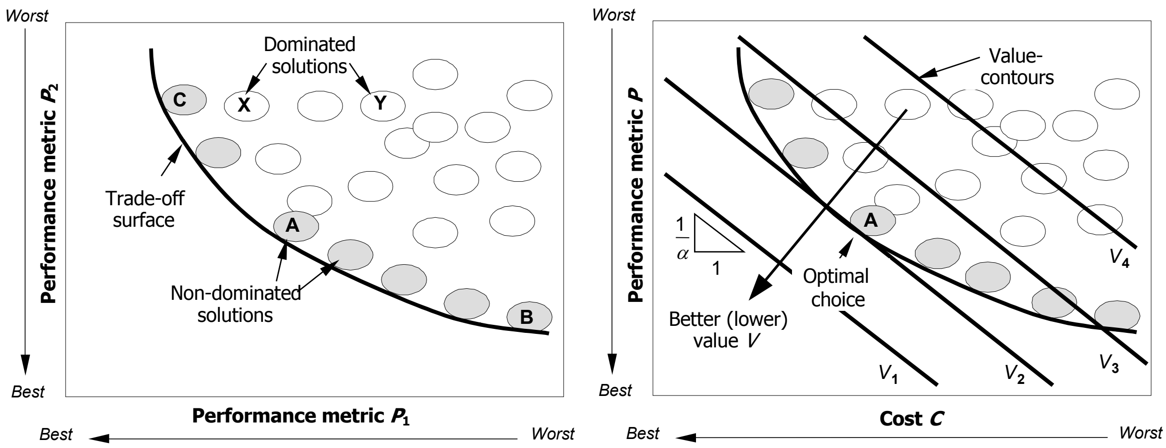


Figure 1. (a) [left] A trade-off plot for performance metrics $P_1$ and $P_2$, showing dominated and non-dominated solutions. The non-dominated solutions lie on the trade-off surface. (b) [right] Contours of value $V$ with slope $-1/\alpha$, superimposed on the trade-off plot. Solution A has the lowest value of $V$ for the chosen value of the exchange constant $\alpha$.

The trade-off surface identifies the subset of solutions that offer the best compromises between the objectives. However, designers ultimately need to select a single solution. One of the most common ways in which to make such a choice is to aggregate the various objectives into a single figure of merit. A composite objective function is formulated such that the minimum of the function defines the most preferable solution. To do this a locally linear utility function, $V$, is defined [1, 10]:

$$V = \sum_{i=1}^{M} \alpha_i P_i \quad (1)$$

where $M$ is the number of objectives. This allows a local minimum to be found. When the search space is large, it is necessary to recognise that the values of the exchange constants $\alpha_i$ may themselves depend on the values of the performance metrics $P_i$.

The utility function reflects the preferability, or utility, or value (equivalent terms) of each solution. The $\alpha$'s are called *exchange constants* (or, equivalently, *utility constants* or *scaling constants*); they convert the units of performance into the unit of utility, $V$, which is usually that of currency ($). The exchange constants are defined by

$$\alpha_i = \left( \frac{\partial V}{\partial P_i} \right)_{\forall P_j, j \neq i} \tag{2}$$

That is, they measure the change in utility for a unit change in a given performance metric, all others being held constant. For instance, if the performance metric $P_1$ is mass $m$ (to be minimised), $\alpha_1$ is the change in utility $V$ associated with unit increase in $m$. The best solution is the one with the smallest value of $V$, which, with properly chosen values of the exchange constants, now correctly balances the conflicting objectives.

Frequently one of the objectives to be minimised is cost, $C$, so that $P_M = C$. Since we have chosen to measure utility in units of currency, unit change in $C$ gives unit change in $V$, with the result that $\alpha_M = 1$ and equation (1) becomes

$$V = C + \sum_{i=1}^{M-1} \alpha_i P_i \tag{3}$$

If $M = 2$, for a given value of $V$ and the exchange constant $\alpha$, equation (3) defines a relationship between the performance metrics $C$ and $P$. This yields a family of parallel lines each for a given value of $V$, as shown in Figure 1(b). The slope of these utility lines is the reciprocal of the exchange constant $-1/\alpha$. The best solution is that at the point at which a utility line is tangential to (a convex part of) the trade-off surface. Here solution A is the best choice.

## 2.2 Values for the exchange constants, $\alpha_i$

An exchange constant is a measure of the utility, real or perceived, of a performance metric. Its magnitude and sign depend on the application. Thus the utility of weight saving in a family car is small, though significant; in aerospace it is much larger. The utility of heat transfer in house insulation is directly related to the cost of the energy used to heat the house; that in a heat-exchanger for power electronics can be much higher. The utility of performance can be real, meaning that it measures a true saving of cost, energy, materials, time or information. But utility can, sometimes, be perceived, meaning that the consumer, influenced by scarcity, advertising or fashion, will pay more or less than the true value of these metrics.

In many engineering applications the exchange constants can be derived approximately from technical models for the life-cost of a system. Thus the utility of weight saving in transport systems is derived from the value of the fuel saved or that of the increased payload, evaluated over the life of the system (Table 1); the utility of heat transfer can be derived from the value of the energy transmitted or saved by unit change in the heat-flux etc. Approximate exchange constants can sometimes be derived from historical pricing-data [10, 11]; thus the utility of weight saving in bicycles can be estimated by plotting the cost $C$ of bicycles against their mass $m$, using the slope $(-dC/dm)$ as a measure of $\alpha$. Finally, exchange constants can be found by interviewing techniques [12, 13], which elicit the utility to the consumer of a change in one performance metric, all others being held constant.

Table 1. Exchange constants $\alpha$ for structural components of transport systems [8].

| Sector: Transport systems | Basis of estimate | Exchange constant US$/kg |
|---|---|---|
| family car | fuel saving | 0.5-1.5 |
| truck | payload | 5-10 |
| civil aircraft | payload | 100-500 |
| military aircraft | payload, performance | 500-1000 |
| space vehicle | payload | 3000-10000 |

## 2.3 How do exchange constants influence choice?

The previous section shows that exchange-constant values depend on the application, and that each application is associated with a characteristic range of exchange-constant values. The discreteness of the search space for material selection means that a given solution on the trade-off surface, such as A in Figure 1(b), is optimal for a certain range of values of $\alpha$; but outside this range another solution becomes the optimal choice. Figure 2 illustrates this. For simplicity solutions have been moved so that, in this figure, only three are potential optima. The remaining non-dominated solutions (D-H) are now on concave sections of the trade-off surface. For $\alpha < 0.1$, C is the optimum; for $0.1 < \alpha < 10$, A is the best choice; and for $\alpha > 10$, it is B. This information is captured in the bar on the right of the figure representing the range of values of $\alpha$, subdivided at the values at which a switch of optimum occurs, and labelled with the solution that is optimal in each range.

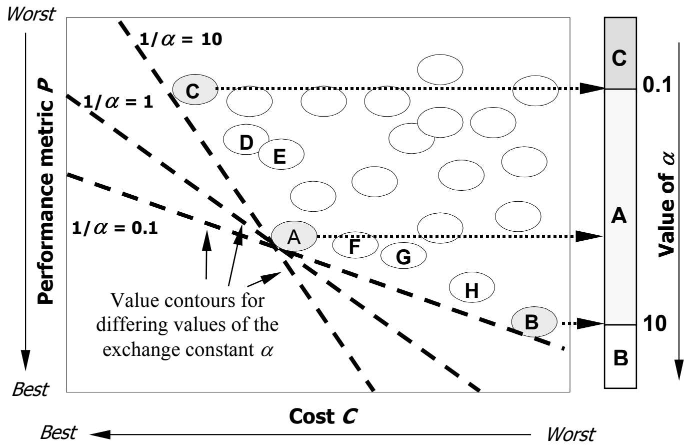


Figure 2. The switches in optimal choice as the exchange constant increases from below 0.1 to above 10. The band at the right shows the range of values of $\alpha$ for which a given solution is optimal.

This suggests a way of extending this form of visualisation to three objectives. It is illustrated in Figures 3 and 4. Figure 3 shows a hypothetical trade-off surface for three performance metrics, one of which is cost. Utility is defined by

$$V = C + \alpha_1 P_1 + \alpha_2 P_2 \tag{4}$$

The two exchange constants $\alpha_1$ and $\alpha_2$ relate $P_1$ and $P_2$ to cost, $C$. The segments of the bar in Figure 2 now become areas, each defining the range of $\alpha_1$ and $\alpha_2$ values for which a given solution that lies on the $P_1$-$P_2$-$C$ trade-off surface is optimal. Remembering that a given application is characterised by given ranges of $\alpha_1$ and $\alpha_2$, such a figure, a hypothetical example of which is shown in Figure 4, offers a way of identifying the best solution for a given application. This reasoning can also be inverted: if a new material (such as H in Figure 4) lies on a convex part of the surface, it will appear somewhere on this diagram; its position suggests applications for which it might be suited.

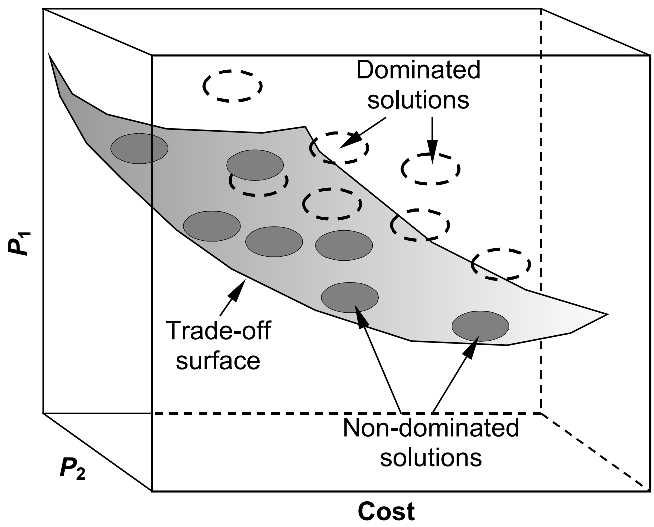


Figure 3. A trade-off surface for three objectives, one that of minimising cost.

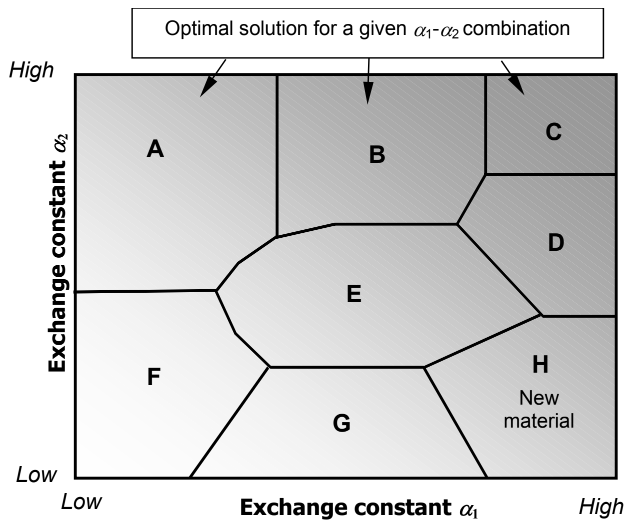


Figure 4. The exchange-constant band shown on the right of the two-objective plot in Figure 2 becomes for three objectives an exchange-constant chart, as shown here.

We have developed an algorithm for identifying the Pareto set for an arbitrary number of objectives and for plotting charts like Figure 4 for any three of them. This is described next.

## 2.4 The algorithm

Consider the following example. We define performance metrics $P_1$, $P_2$ and $P_3$ for solutions to a design problem based on the alternative material choices A-E. The selection procedure starts by seeking the non-dominated solutions. We adopt the criteria proposed by [1] to determine dominance and non-dominance (assuming all performance metrics are to be minimised):

**If** all the performance metrics $P_i$ for material 'A' are equal to or less than those for material 'B',

**and** there is at least one performance metric for material 'A' that is less than that for material 'B' (neither dominates when all their metrics $P_i$ are identical),

**then** material 'A' dominates material 'B'.

The set of comparisons can be shown as a square matrix, the *dominance matrix*, of size equal to the number of materials in the database. Its cells contain the *dominance index*, which is defined as the number of performance metrics $P_i$ for that row's material that have values equal to or less than those for the material of the corresponding column. If the dominance index equals the number of objectives and all the performance metrics are not identical, the material corresponding to that row dominates the one for the corresponding column. The non-dominated materials are then identified as the materials with columns that do not contain any dominance indices that equal the number of objectives.

This sounds simple, but it can be computationally intensive for large material databases. Deterministic methods can be used to reduce the number of steps, as illustrated in Figure 5 for the hypothetical data set given in Table 2.

Table 2. Hypothetical data set for the example shown in Figure 5.

| Material | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|
| A | 2.0 | 5.0 | 3.0 |
| B | 10.0 | 8.0 | 7.0 |
| C | 6.0 | 4.0 | 8.0 |
| D | 2.0 | 5.0 | 3.0 |
| E | 5.0 | 4.0 | 2.0 |

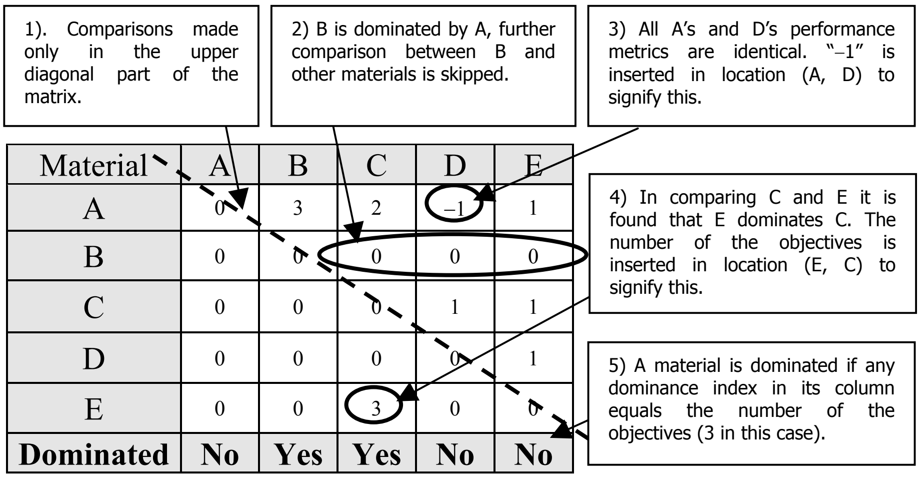


Figure 5. An example of a dominance matrix and the shortcuts used in constructing it.

Once the non-dominated set is identified, a utility function is used to find a final solution. An optimal solution, for a given set of exchange constants, is found by comparing the utility function for each non-dominated solution, remembering that, in the convention adopted here, the solution with the lowest $V$ is the best choice. By stepping through values of the exchange constants and identifying the optimal non-dominated solution for each set of values, a chart like that of Figure 4, showing the ranges of exchange constants for which each non-dominated solution is optimal, can be constructed.

The boundaries separating the fields in this chart are *equal utility boundaries*; on the boundaries two non-dominated solutions have the same, lowest, utilities simultaneously. To locate these boundaries, the non-dominated solutions are equated in pairs, defining the trajectory along which $V_i = V_j$, i.e. where

$$V_i - V_j = (\Delta C)_{i-j} + \alpha_1 (\Delta P_1)_{i-j} + \alpha_2 (\Delta P_2)_{i-j} = 0 \qquad (5)$$

$\forall\, i, j \in N, i \neq j$, $N$ being the number of non-dominated materials.

This algorithm has been applied to material selection problems using a database of properties for 3,000 materials [14]. Typical output is as shown in Figure 6, which is the solution to the 3-objective problem detailed in Section 3, requiring the minimisation of mass and cost while maximising heat transfer. On the x-axis is the exchange constant for mass and cost, $\alpha_1$; on the y-axis is that for heat transfer and cost, $\alpha_2$. The material in the bottom left corner (cast iron) is the best choice when minimising cost is paramount, but, as the importance attached to low mass and/or high heat transfer increases, the optimal choice changes. Note that exact values for the exchange constants for a given application, which are often difficult to establish, are not required: a given material is generally optimal over substantial ranges of $\alpha_1$ and $\alpha_2$ values. In addition, sensitivity analysis, which enables the designer to scrutinise the effects of varying his/her preference articulation (by varying the values of the exchange constants), can be performed very easily using such a chart.

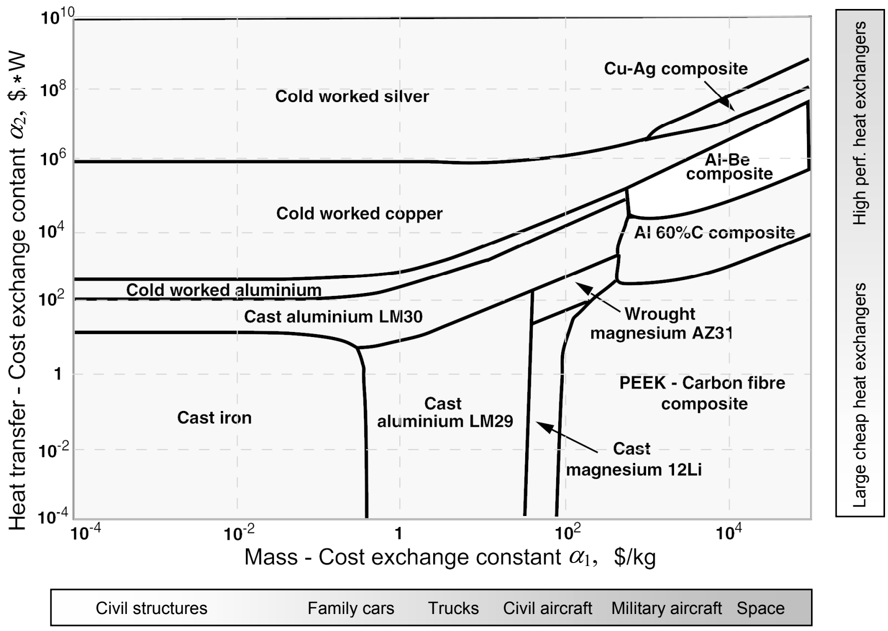


Figure 6. An exchange-constant chart for a design problem with three objectives: those of minimising mass and cost, and of maximising heat transfer. Details are given in Section 3.

# 3. Case study: Materials for a disk brake calliper

To see in more detail how our method works, a case study is helpful. Figure 7 shows a schematic of a brake calliper for a high performance car. It can be idealised as two beams of length $L$, width $b$ and thickness $h$, locked together at their ends. Each is loaded in bending and is exposed to high temperatures. The lower schematic represents one of the beams; its length $L$ is given, and $b$ scales with $h$ such that $b = \gamma h$, where $\gamma$ is a shape-factor. The beam stiffness $S$ is critical: if it is inadequate the calliper will flex, impairing braking efficiency and allowing vibration. Its ability to transmit heat, too, is important since some of the heat generated in braking must be conducted out through the calliper.

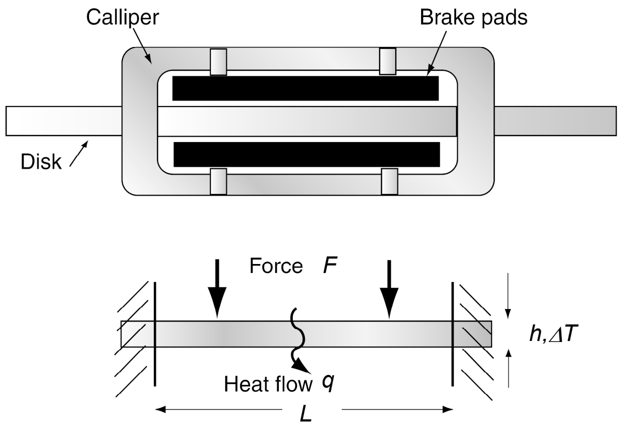


Figure 7. A schematic of a brake calliper. The long arms are loaded in bending.

There are three objectives: minimising mass, maximising heat-transfer and minimising cost. The mass of the calliper scales with that of one of the beams, described by the equation

$$m = L\,b\,h\,\rho = L\gamma h^2\,\rho \text{ (kg)} \tag{6}$$

where $\rho$ is the density of the material of which it is made. Heat transfer rate $q$ depends on the thermal conductivity $\lambda$ of the beam material:

$$q = L\,b\,\lambda\frac{\Delta T}{h} = L\,\gamma\,\lambda\,\Delta T \text{ (W)} \tag{7}$$

where $\Delta T$ is the temperature difference between the surfaces. Finally, the material cost depends on the mass $m$ and the cost per unit mass, $C_m$, of the beam material:

$$C = C_m\,m \text{ (\$)} \tag{8}$$

The quantities $L$, $\gamma$ and $\Delta T$ are specified. The only free variable is the thickness $h$. But there is a constraint: the calliper must be stiff enough to ensure that it does not flex or vibrate excessively. To achieve this we require that

$$S = \frac{C_1 E I}{L^3} = \frac{C_1 E\gamma h^4}{12\,L^3} \geq S^* \tag{9}$$

where $S^*$ is the desired stiffness, $E$ is Young's modulus, $C_1$ is a constant which depends on the load distribution and $I = bh^3/12 = \gamma h^4/12$ is the second moment of area of the beam. Thus

$$h^4 = \frac{12\,S\,L^3}{C_1 E\gamma} \tag{10}$$

Inserting this in equation (6) and rearranging gives an equation for the performance metric $P_1$:

$$P_1 = m \geq \left( \frac{12\gamma S^*}{C_1} \right)^{1/2} L^{5/2} \left( \frac{\rho}{E^{1/2}} \right) \text{(kg)} \tag{11}$$

Rearranging equation (7) gives:

$$P_2 = \frac{1}{q} = \frac{1}{\gamma L \lambda \Delta T} \; (\mathrm{W}^{-1}) \tag{12}$$

To identify optimal solutions we must combine equations (8), (11) and (12) in the utility function

$$V = C + \alpha_1 P_1 + \alpha_2 P_2 \; (\$) \tag{13}$$

where $\alpha_1$ has units of \$/kg and $\alpha_2$ of \$/(W$^{-1}$). Figure 6, introduced earlier, is an exchange-constant chart derived from equation (13), showing how the optimal choice changes as the exchange constants vary. The materials with the highest performance (and cost) lie towards the top right corner. When low mass and high heat-transfer are very highly valued, as they are in Formula 1 racing, these are the optimal choices. Among them is a novel Al-Be powder metallurgy alloy that combines the high conductivity of aluminium with the low density and exceptional stiffness of beryllium – but at a price. A brake calliper designed for Ferrari Racing is made of precisely this alloy [15].

Note how the method points to applications for new materials, of which the Al-Be composite is an example. If it were not in the database, the materials that surround it in Figure 6 would occupy its space. Add it to the database and it appears at the position shown. Its "co-ordinates" in $\alpha_1$-$\alpha_2$ space identify applications in which stiffness at low mass and high heat transfer are highly valued, and identify the materials that compete with it for this market.

# 4. Conclusions

Almost all material selection problems require that a compromise be sought between some metric of performance and cost. Trade-off methods using utility functions allow optimal solutions to be found for two objectives, but for three it is harder. Here we develop and demonstrate a method for dealing with three objectives. It involves an algorithm for identifying the trade-off surface for a multi-objective problem, and a novel way of showing how the optimal choice depends on the relative value attached to each objective. In a single chart the designer is presented with an overview of every possible outcome (for every set of preferences) for a given application, and is readily able to see the effects of varying their preferences. Thus the new approach provides additional insights into material selection problems, which are not available using other material selection methods. The method lends itself to implementation in software as part of a design tool to guide material selection.

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