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GEOMETRIC MODELING OF MECHANICAL PARTS AFFECTED BY SHAPE ERROR

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1. Introduction

Many software analysis tools are available for product simulation that enable designers to perform a wide range of engineering analysis such as thermal, structural, dynamic, aerodynamic and similar. These simulations are usually performed using digital mock-ups where the component, which is being studied, is described in terms of nominal dimensions and shape.

Nevertheless it is well known that all the surfaces of a real component deviate from the ideal, with respect to shape, location, size and orientation. Such deviations are recognised to be the result of the entire production process, including effects as material properties, the static and dynamic behaviour of machine tools, thermal effects, workpiece positioning errors and others [Henke 1999].

The interaction between geometric shape error and environment performances subject to it, is generally complex and difficulty predictable by deterministic method.

Some authors [Fowlkes 1995] noted how, for *robust design*, the effects of the uncertainties due to shape errors should be taken into account from the outset of the entire deign process.

The CAD generation of solid models affected by errors is necessary for the realisation of experiment plains in order to evaluate, at priori, the performances and reliability of tested component. In the work of J.S. Mira [Mira 2002] geometric errors are simulated to analyse the consequence of macro geometric variations of the thrust bearing kinematics joint. Bianconi [Bianconi 2003] describes how kinematics shape errors can be defined and how they can be applied to the model surface.

In this article we describe a method for automatic generation of CAD models affected by shape errors to be used in specific CAE environments for simulation in presence of shape errors. In particular a stochastic model for line shape error generation is proposed. The model is suited for the generation of a random error conditioned by some requirements. In order to analyse the possibility to control the main characteristics of the geometric shape error, some experiments has been conducted for different values of the model parameters. The results of these experiments have been used to define a function that correlates the model parameters with the technical parameters. The results obtained are presented and critically discussed.

2. Geometric error

The development of a design error simulation system requires, preliminarily, a consistent design error specification. The concept of the geometric tolerances, in the ANSI Y14.5M and in ISO 1101 standards, is expressed graphically as a distance between geometric entities. The stochastic nature of the geometric error generation implies a design error specification that contemplates the possibility to control the main characteristics of the error pattern.

The geometric error results of the joint effect of the two different geometric components: random error component and deterministic error component. These two components are combined together so that the geometric error could be assumed as random generated under the conditions imposed by some constraints that reproduce the typical deterministic affect due to the manufacturing process. The main characteristics of a geometric profile affected by a shape error are:

- The geometric profile is a continuous autocorrelate curve;
- The geometric profile is in statistical equilibrium about a nominal geometry;
- The surface irregularities are characterised by an expected value of their spacing;
- The expected value of the maximum height of the geometric profile does not exceed the value of the geometric tolerance *t*.

The technical parameters assumed in this work to control the generation process of the shape error of a line, are the root mean square of the point of the line from its nominal shape (Y_{RMS}), and the mean spacing of profile irregularities (S).

$$Y_{RMS} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2} \qquad S = \frac{1}{n}\sum_{i=1}^{n} s_i \tag{1}$$

In the above formula s_i is the space of the i^{th} irregularity. The vector $\mathbf{t}\{t, Y_{RMS}, S\}$ defines the straightness geometric tolerance. Its components represent the expected values of the design parameters of the geometric error profile to be generated. This definition of the geometric tolerance contemplates some elements necessary to virtually generate a geometric part affected by an error.

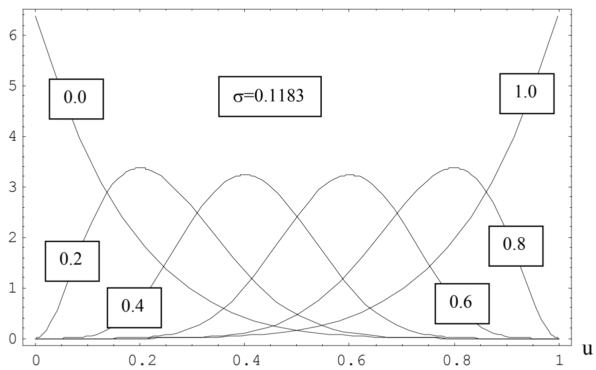

Figure: 1. The beta density functions for different values of the mode values and for a fixed value of the standard deviation (0.1183).

2.1 Beta distribution

Beta distribution is a very flexible Probability Density Function (PDF) suited for modelling various stochastic process. The unit beta PDF is given as follows:

$$f(u,\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} u^{\alpha-1}(1-u)^{\beta-1}, 0 \le u \le 1 \tag{2}$$

$\Gamma()$ is the Euler gamma function. The main characteristic of the beta distribution is that its shape can be changed by modifying the shape parameters α and β. The values of the shape parameters, in this work, are assumed greater then 1 so that one-maximum beta distributions can be generated. The value of the mode (m), standard deviation (σ) and mean (μ) of the beta distribution are:

$$m = \frac{\alpha-1}{\alpha+\beta-2} \quad \sigma = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \quad \mu = \frac{\alpha}{\alpha+\beta} \tag{3}$$

In order to control the random generation process the beta function, in this work, has been defined, as a function of the two statistical parameters σ and *m,* by solving the equations (2) and (3), eliminating the variables α and β. In figure 1 the beta function are shown as functions of *m* for a fixed value of σ.

2.2 Stochastic model of the geometric error

A normalised model of the geometric error generation of a straight line has been defined. The error generation process is controlled so that the error profile tends to remain in a unitary referring space. An autoregressive stochastic model generates the geometric error step by step at constant intervals. It is a stationary model, which generates a geometric profile in equilibrium about a constant mean level (mean line of the profile). In this model the current value of the profile ordinate ξ_i is expressed as a linear function of the previous value of the error.

$$\xi_i = \xi_{i-1} + \Delta\xi_i \quad \text{Where } \Delta\xi_i = \Delta\xi_{i-1} + \varepsilon_i(\xi_{i-1}, \Delta e, \sigma) \tag{4}$$

The additional term ε_i is a random number generated at each iteration step defined as follows:

$$\varepsilon_i = \left(u(\sigma, m) - \frac{1+2\xi_{i-1}}{2} \right)\Delta e \quad m = \begin{cases} \xi_{i-1} \geq 1/2 \Rightarrow m = 1 \\ -1/2 < \xi_{i-1} < 1/2 \Rightarrow m = \xi_{i-1} + 1/2 \\ \xi_{i-1} \leq -1/2 \Rightarrow m = 0 \end{cases} \tag{5}$$

The random number u ($0 \geq u \geq 1$) in equation (5) is generated with a beta probability density function which shape is modified as a function of the current value of ξ_{i-1}. The beta functions are generated for an assigned value of the standard deviation σ, which is one of the parameters to be assumed to generate a specific type of geometric error. The expected value of the $\Delta\xi_i$ correction is also controlled by the Δe parameter.

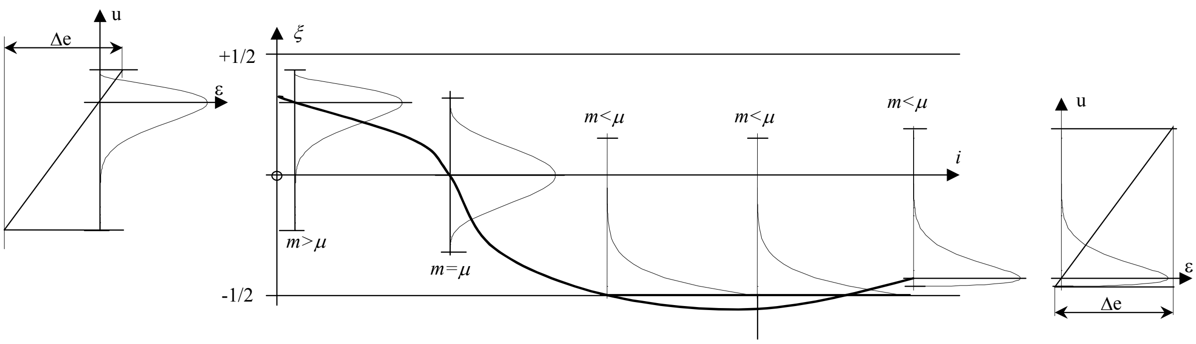

Figure 2. Normalised error profile and probability distributions for ε_i random generation.

The $\Delta\xi_i$ correction, defined by equation (5), is assumed so that for $u_i(\sigma,m)=m \Rightarrow \varepsilon_i=0$. For this reason in each generation step the maximum probability density is that the $\Delta\xi_i$ does not change ($\Delta\xi_i = \Delta\xi_{i-1}$). In any cases, when the current position of the geometric profile is out of the nominal position ($\xi_{i-1} \neq 0$), the probability that the absolute value of $\Delta\xi_i$ decrease in greater than it increase. The generated error remains about the constant mean defined by the nominal geometry. In figure 2 an example of the error profile is shown with some examples of the beta functions for different values of the current profile position.

2.3 Geometric modelling of the errors

The configuration of the geometric error generation system is sketched in figure 3. The geometric error can be generated once the independent parameters **s** $\{\sigma, \Delta e, \tau, \Delta x\}$ have been defined. At this purpose the design error specification **t**$\{t, Y_{RMS}, S\}$ must be converted in the input parameters of the error generator: $\mathbf{s} = \mathbf{Tt}$. The transformation operator **T** is a complex operator that performs a non-linear map between design error specification to error modelling parameter **s**. In order to realise this transformation process, in this work, an artificial neural network has been used. It is a typical back propagation neural network which structure consists of two hidden layers and sigmoidal activation function of the hidden nodes. 80 couples of simulated data ($\mathbf{s}_j$, $\overline{\mathbf{t}}_j$) have been used as examples to train the artificial neural network and define the transformation operator **T**. Each couple of data has been obtained by evaluating the mean values($\overline{\mathbf{t}}_j$), of the $\mathbf{t}_j$ parameters, of 30 profiles generated all in the same conditions ($\mathbf{s}_j$).

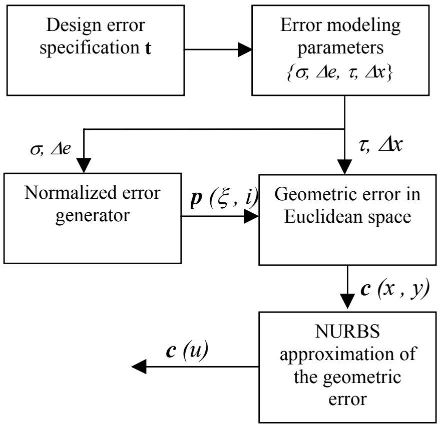

Figure 3. Geometric error generation system

The normalised error can be converted in the profile error, in the Euclidean space, by multiplying the value of the normalised ordinate for a factor τ ($y_i = \xi_i \tau$) and by converting the sequence position i ($x_i = i\Delta x$). At the end, in order to obtain the geometric profile in a typical CAD format, the set of points **c***(x,y),* which defines the profile error, are approximated by a NURBS curve.

3. Error simulations and tests

The methods has been verified in simulating geometric error for different values of the parameters **t**. In all the tested cases the parameters of the simulated error do not diverge more than 25% from the expected values. These differences do not represent prejudicial element in error simulation and they are the consequence of the stochastic nature of the generation process.

In order to verify the model capability in geometric error simulation, an acquired geometric error has been assumed as a target geometric error. The target has been obtained by acquiring a profile of a surface machined by a milling process. The acquired profile has been processed by a gaussian filter (cut-off 0.8-mm) to isolate the geometric error component from the roughness. In figure 4 the geometric error measured and the filtered profile are shown. The geometric error chosen represent an interesting test due to its particular irregular trend.

Two profiles error has been generated to simulate the target geometric error. The simulation 1 does not present appreciable difference in the technical parameters from the target profile. Simulation 2 has a different value of the *t* parameter Table1. In figure 5 the profiles error generated (simulation 1 and simulation 2) are compared with that target profile. Although simulation 1 and the target profile present almost similar technical parameters, their shapes are quite different. In order to investigate the nature of the geometric error simulated and the capability of the proposed model to reproduce the harmonic components, some specific analysis has been conduced. The autocorrelation of each profile are estimated and the results are shown in figure 6. The autocorrelation evidenced that the simulation 2 best fit the characteristic of the experimental profile. This is also evidenced from the FFT (Fast Fourier Transform) analysis of the profiles which results are quoted in table 2.

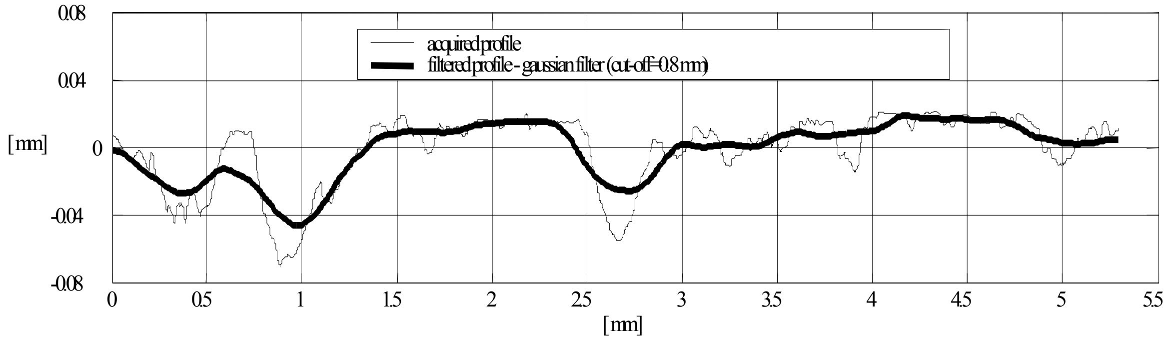

Figure 4. Geometric error measured and filtered profile

Table 1.

Profile	t	Y_{RMS}/t	S	e_{Ry}	e_R	e_S
Experimental	0.07801	0.2836	0.6203			
Simulation 1	0.06871	0.2731	0.58011	11.92%	3.65%	6.48%
Simulation 2	0.0591	0.2402	0.7187	24.33%	15.3%	15.86%

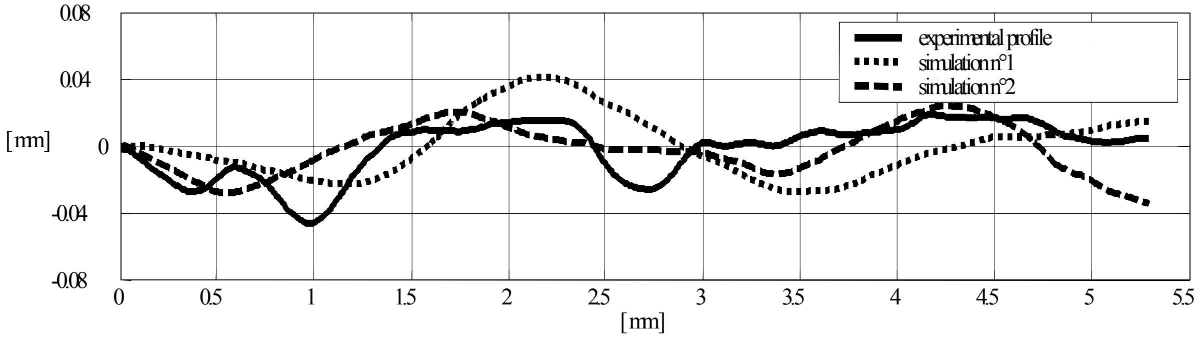

Figure 5. Simulated geometric errors and experimental profile

The proposed model is not suited to simulate exhaustively very indented profiles. The proposed method perform best results in simulating errors, or theirs components, characterised by a smooth trend, which is typical of many practical applications.
In figure7 an application of the proposed model in the linear error simulation of the axes of a mechanical component is shown. In this example the error has been amplified.

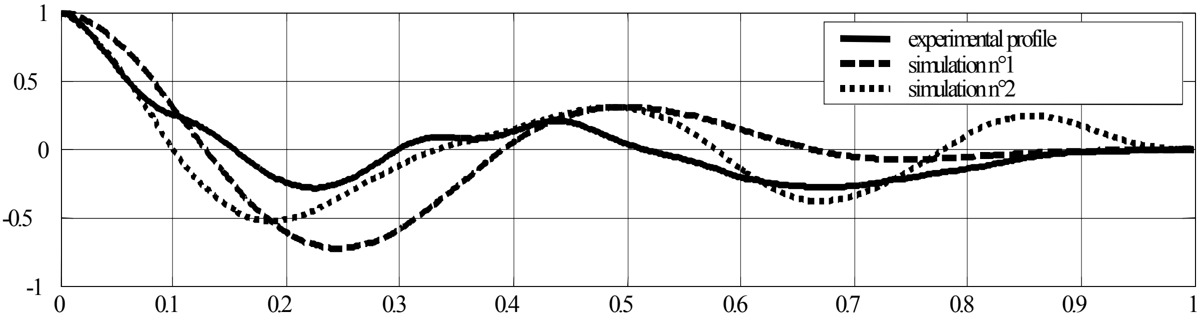

Figure 6. Autocorrelation curves of the geometric error profiles

Table 2.

Profile	frequency			amplitude		
	1st harmonic	2nd harmonic	3rd harmonic	1st harmonic	2nd harmonic	3rd harmonic
Experimental	0.7472	2.6151	3.3623	0.01549	0.00601	0.00393
Simulation 1	0.748	1.8699		0.2389	0.0287	
Simulation 2	0.748	1.9439	2.8659	0.16815	0.07695	0.03373

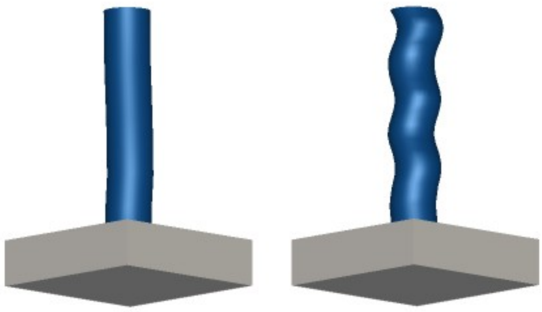

Figure 7. Example of a simple application.

4. Conclusions

The results of the first step of a research activity concerning geometric error simulation are presented. An original algorithm for geometric error generation is illustrated and the results of its implementation are reported. Neural network has been used to correlate parameter of the error profile with technical parameters that characterise the geometric error. Some tests have been performed and the proposed approach proves to be useful to reproduce the assigned technical parameters (maximum height of the error profile, root-mean-square deviation of the profile and mean spacing of profile irregularities). The proposed method has been analysed in the reconstruction of real profiles that have been experimentally obtained. Although the tests have shown that the assumed technical parameters can be reproduced very well, a limit is evidenced in reproducing complex irregular trends such those typically observed in roughly machined surfaces. In any case the proposed model show promising capability to reproduce the smooth components of the geometric error appointing the sharp irregularities simulation to more appropriate methods.

The proposed method can be used in the analysis of many industrial applications: all those where the shape error simulation represent an important factor. A promising utilisation is in Monte Carlo simulations to generate the instances to be analysed. Some examples of applications, are: contact problems, tolerance analysis of complex assembly, aesthetic verification of shiny surfaces with not wanted waviness, simulation of component with higher kinematics pairs (cams, gears) etc.

Next goal of this research activity is to extend the proposed method to generate geometric shape errors of surfaces.

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